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Problems:

- **Compute the equilibrium temperature of the surface and of the atmosphere.**
- **Investigate the effects of *changing parameters* on the temperature.**

Exercise:

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Assume that the atmosphere can be regarded as a thin layer with an absorbtivity of $a_S = 0.1$ for shortwave (solar) radiation and $a_L = 0.8$ for longwave (terrestrial) radiation.

Assume the Earth's *albedo* is $A = 0.3$ and the *solar constant* is $F_{\rm solar} = 1370\,\mathbf{W\,m}^{-2}$.

Assume that the earth's surface radiates as a blackbody at all wavelengths.

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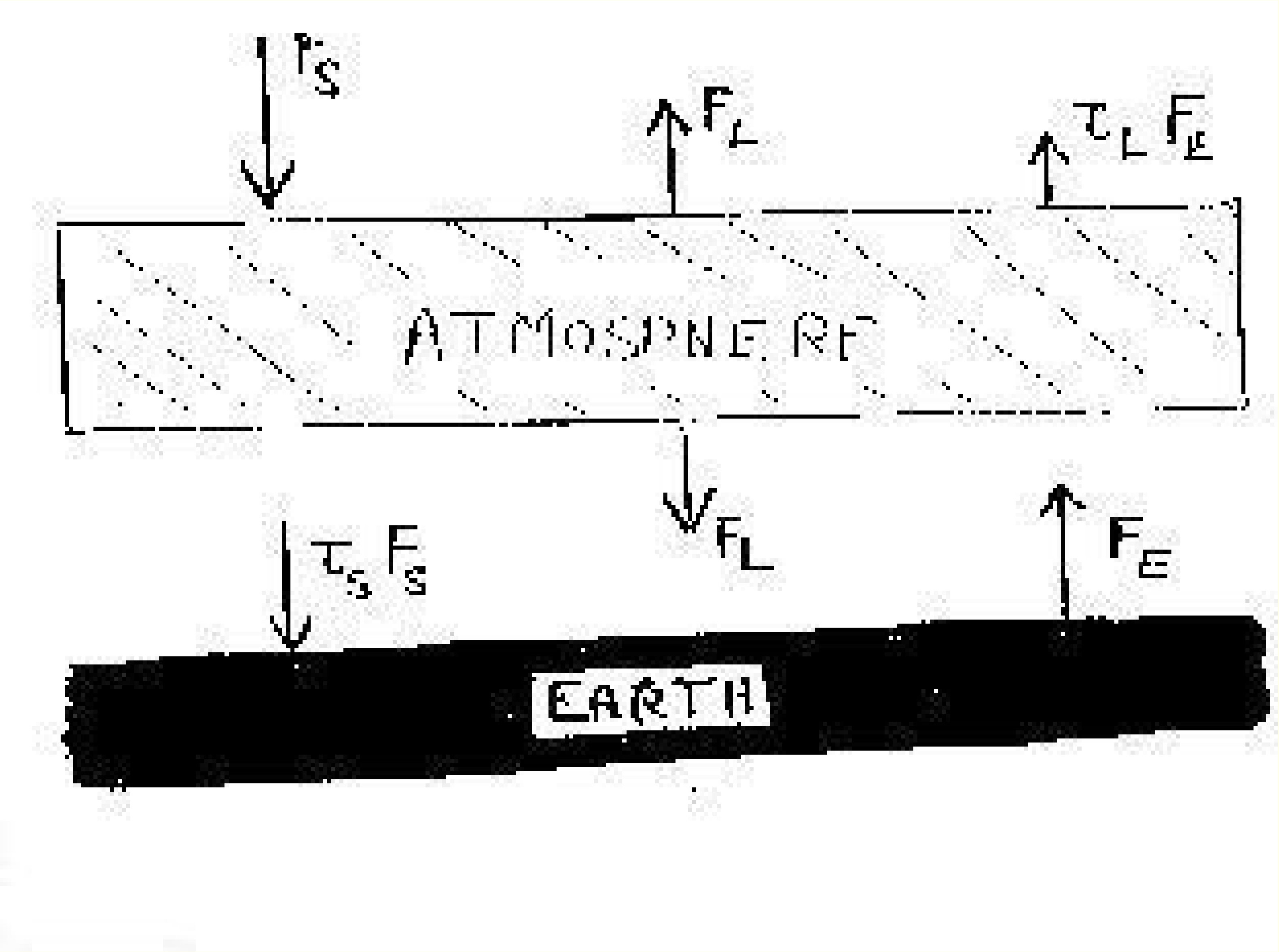
Calculate the radiative equilibrium temperature T_E of the surface and the *sensitivity of T_E to changes* in the following parameters:

- **Absorbtivity of the atmosphere to shortwave radiation**
- **Absorbtivity of the atmosphere to longwave radiation**
- **Planetary albedo**
- **Solar constant**

Solution:



Solution:

The net solar irradiance F_S *absorbed* by the earth-atmosphere system is equal to the solar constant *reduced by* the albedo and by the areal factor of four:

$$F_S = \left(\frac{1-A}{4}\right) F_{\mathrm{solar}} = \frac{0.7}{4} \times 1370 = 240\,\mathrm{W\,m}^{-2}$$

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We define the *transmissivity* as $\tau_S = 1 - a_S$.

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Let F_E be the longwave <u>flux emitted upwards by the surface</u>.

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Let F_E be the longwave <u>flux emitted upwards by the surface</u>.

Since the absorbtivity for terrestrial radiation is $a_L = 0.8$, the *longwave transmissivity* is $\tau_L = 1 - a_L = 0.2$.

Thus, there results an upward flux at the top of the atmosphere of $\tau_L \, F_E$.

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Radiative balance at the surface (upward flux equal to downward flux) gives:

$$F_E = \tau_S F_S + F_L$$

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The upward and downward fluxes at the top of the atmosphere must also be in balance, which gives us the relation

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To find F_E and F_L, we solve the simultaneous equations

$$\begin{aligned} F_E - F_L &= \tau_S F_S \\ \tau_L F_E + F_L &= \quad F_S \end{aligned}$$

Repeat: to find F_E and F_L, we must solve

$$\begin{aligned} F_E - F_L &= \tau_S\, F_S \\ \tau_L\, F_E + F_L &= F_S \end{aligned}$$

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This gives the values

$$F_E = \left(\frac{1+\tau_S}{1+\tau_L}\right) F_S \qquad F_L = \left(\frac{1-\tau_S\tau_L}{1+\tau_L}\right) F_S$$

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Assuming that the Earth radiates like a blackbody, the Stefan-Boltzman Law gives

$$\sigma T_{\text{surface}}^4 = F_E$$

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$$\sigma T^4_{\text{surface}} = \frac{1+\tau_S}{1+\tau_L}\left(\frac{1-A}{4}\right) F_{\text{solar}}$$

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Taking logarithms, we have

$$\log\sigma + 4\log T_{\text{surface}} = \log(1+\tau_S) - \log(1+\tau_L) + \log(1-A) - \log 4 + \log F_{\text{solar}}$$

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Now differentiate:

$$4\frac{dT_{\mathrm{surface}}}{T_{\mathrm{surface}}} = \frac{d\tau_S}{1+\tau_S} - \frac{d\tau_L}{1+\tau_L} - \frac{dA}{1-A} + \frac{dF_{\mathrm{solar}}}{F_{\mathrm{solar}}}$$

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This equation enables us to investigate the sensitivity of the surface temperature to changes in various parameters.

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This equation enables us to investigate the sensitivity of the surface temperature to changes in various parameters.

For reference, let's call this the Blue Equation.

$\star \quad \star \quad \star$

Sensitivity to Shortwave Absorbtion

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So,

$$a_S \Rightarrow a_S + d\,a_S$$
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Clearly, if $d\,a_S > 0$ **then** $d\,\tau_S = -d\,a_S < 0$**.**

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Suppose the transmissivity decreases from 0.9 to 0.8. Then $\tau_S = 0.9$ and $d\tau_S = -0.1$.

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Suppose the transmissivity decreases from 0.9 to 0.8. Then $\tau_S = 0.9$ **and** $d\tau_S = -0.1$**.**

Suppose also that the equilibrium temperature of the Earth with $\tau_S = 0.9$ **is 288 K (as we found above).**

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Then

$$4\,\frac{dT_{\text{surface}}}{288} = \left(\frac{-0.1}{1.9}\right) \qquad \textbf{or} \qquad dT_{\text{surface}} = \left(\frac{-0.1}{1.9}\right)\frac{288}{4} = -3.8\,K$$

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Thus, the assumed increase in shortwave absorbtivity has resulted in a decrease in surface temperature of about 4°C.

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Suppose the longwave transmissivity decreases from 0.2 to 0.1. Then $\tau_L = 0.2$ **and** $d\tau_S = -0.1$**.**

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Suppose the longwave transmissivity decreases from 0.2 to 0.1. Then $\tau_L = 0.2$ **and** $d\tau_S = -0.1$**.**

Suppose once more that the equilibrium temperature of the Earth with $\tau_L = 0.2$ **is 288 K.**

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Thus, the assumed increase in longwave absorbtivity has resulted in an increase in surface temperature of about 6°**C.**

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Suppose once more that the equilibrium temperature of the Earth with $A = 0.3$ is 288 K.

Then

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Thus, the assumed increase in albedo has resulted in an decrease in surface temperature of about 10°C.

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Suppose once more that the equilibrium temperature of the Earth is 288 K.

Again,

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Suppose once more that the equilibrium temperature of the Earth is 288 K.

Then

$$4\,\frac{dT_{\rm surface}}{288} = \left(\frac{0.01\,F_{\rm solar}}{F_{\rm solar}}\right) \qquad \textbf{or} \qquad dT_{\rm surface} = 0.01\times\frac{288}{4} = 0.7\,K$$

Again,

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Suppose once more that the equilibrium temperature of the Earth is 288 K.

Then

$$4\,\frac{dT_{\text{surface}}}{288} = \left(\frac{0.01\,F_{\text{solar}}}{F_{\text{solar}}}\right) \qquad \textbf{or} \qquad dT_{\text{surface}} = 0.01 \times \frac{288}{4} = 0.7\,K$$

Thus, the assumed 1% increase in solar flux has resulted in an increase in surface temperature of less than 1°**C.**

Review of Sensitivities

Review of Sensitivities

- **An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about $4°$C.**

Review of Sensitivities

- **An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about 4°C.**
- **An increase in longwave absorbtivity of 0.1 resulted in an increase in surface temperature of about 6°C.**

Review of Sensitivities

- **An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about 4°C.**
- **An increase in longwave absorbtivity of 0.1 resulted in an increase in surface temperature of about 6°C.**
- **An increase in albedo of 0.1 resulted in an decrease in surface temperature of about 10°C.**

Review of Sensitivities

- **An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about $4°$C.**
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In general, all parameters undergo small changes. Moreover, we have completely neglected the effects of water vapour and liquid water.

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In general, all parameters undergo small changes. Moreover, we have completely neglected the effects of water vapour and liquid water.

The results give an indication of how difficult it is to gauge the consequences for climate of any changes which may occur.

End of §4.5