Climate Sensitivity

We consider climate sensitivity in a very simple context.

- We consider a single-layer isothermal atmosphere.
- We assume the system is in radiative balance.
- We assume the atmosphere is almost transparent to shortwave radiation.
- We assume the atmosphere is relatively opaque to longwave radiation.
- We assume the Earth radiates like a blackbody.

Problems:

- Compute the equilibrium temperature of the surface and of the atmosphere.
- Investigate the effects of *changing parameters* on the temperature.

Exercise:

Assume that the atmosphere can be regarded as a thin layer with an absorbtivity of $a_S = 0.1$ for shortwave (solar) radiation and $a_L = 0.8$ for longwave (terrestrial) radiation.

Assume the Earth's ***albedo*** is $A = 0.3$ and the ***solar constant*** is $F_{\text{solar}} = 1370\,\mathbf{W\,m}^{-2}$.

Assume that the earth's surface radiates as a blackbody at all wavelengths.

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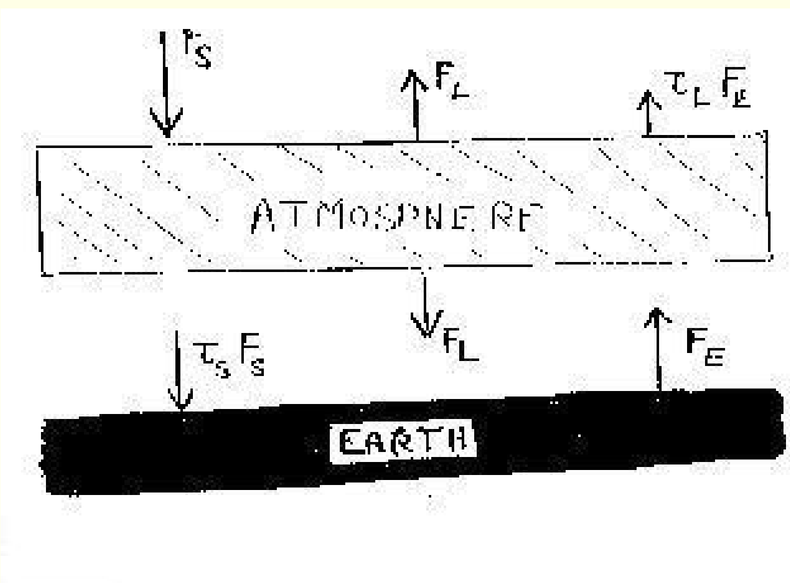
Calculate the radiative equilibrium temperature T_E of the surface and the ***sensitivity of T_E to changes*** in the following parameters:

- Absorbtivity of the atmosphere to shortwave radiation
- Absorbtivity of the atmosphere to longwave radiation
- Planetary albedo
- Solar constant

Solution:



Solution:

The net solar irradiance F_S ***absorbed*** by the earth-atmosphere system is equal to the solar constant ***reduced by*** the albedo and by the areal factor of four:

$$F_S = \left(\frac{1-A}{4}\right) F_{\text{solar}} = \frac{0.7}{4} \times 1370 = 240\,\text{W}\,\text{m}^{-2}$$

Therefore, the incoming flux of solar radiation at the top of the atmosphere, averaged over the whole Earth, is $F_S = 240\,\mathbf{W\,m}^{-2}$.

The ***absorbtivity*** for solar radiation is $a_S = 0.1$.
We define the ***transmissivity*** as $\tau_S = 1 - a_S$.

The downward flux of short wave radiation at the surface is the incoming flux multiplied by the transmissivity, $\tau_S F_S$.

Let F_E be the longwave flux emitted upwards by the surface.

Since the absorbtivity for terrestrial radiation is $a_L = 0.8$, the ***longwave transmissivity*** is $\tau_L = 1 - a_L = 0.2$.

Thus, there results an upward flux at the top of the atmosphere of $\tau_L F_E$.

Let F_L be the long wave flux emitted upwards by the atmosphere; this is also the long wave flux emitted *downwards*.

Thus, the total downward flux at the surface is $\tau_S F_S + F_L$

Radiative balance at the surface (upward flux equal to downward flux) gives:

$$F_E = \tau_S F_S + F_L$$

The upward and downward fluxes at the top of the atmosphere must also be in balance, which gives us the relation

$$F_S = \tau_L F_E + F_L$$

To find F_E and F_L, we solve the simultaneous equations

$$\begin{aligned} F_E - F_L &= \tau_S F_S \\ \tau_L F_E + F_L &= \quad F_S \end{aligned}$$

Repeat: to find F_E and F_L, we must solve

$$\begin{aligned} F_E - F_L &= \tau_S F_S \\ \tau_L F_E + F_L &= \quad F_S \end{aligned}$$

This gives the values

$$F_E = \left(\frac{1+\tau_S}{1+\tau_L}\right) F_S \qquad F_L = \left(\frac{1-\tau_S\tau_L}{1+\tau_L}\right) F_S$$

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Assuming that the Earth radiates like a blackbody, the Stefan-Boltzman Law gives

$$\sigma T^4_{\rm surface} = F_E$$

using the expressions derived for F_S and F_E, this is

$$\sigma T^4_{\rm surface} = \frac{1+\tau_S}{1+\tau_L}\left(\frac{1-A}{4}\right) F_{\rm solar}$$

Again,

$$\sigma T^4_{\rm surface} = \frac{1+\tau_S}{1+\tau_L}\left(\frac{1-A}{4}\right) F_{\rm solar}$$

Taking logarithms, we have

$$\log\sigma + 4\log T_{\rm surface} = \log(1+\tau_S) - \log(1+\tau_L) + \log(1-A) - \log 4 + \log F_{\rm solar}$$

Now differentiate:

$$4\frac{dT_{\rm surface}}{T_{\rm surface}} = \frac{d\tau_S}{1+\tau_S} - \frac{d\tau_L}{1+\tau_L} - \frac{dA}{1-A} + \frac{dF_{\rm solar}}{F_{\rm solar}}$$

This equation enables us to investigate the sensitivity of the surface temperature to changes in various parameters.

For reference, let's call this the Blue Equation.

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Sensitivity to Shortwave Absorbtion

Suppose that some change causes an ***increase in the absorbtion*** of ***solar radiation*** by the atmosphere.

For example, an increase in ozone concentration in the stratosphere would result in greater absorbtion of incoming solar radiation.

So,

$$\begin{aligned} a_S &\Rightarrow a_S + d\,a_S \\ \tau_S &\Rightarrow \tau_S + d\,\tau_S \end{aligned}$$

Clearly, if $d\,a_S > 0$ then $d\,\tau_S = -d\,a_S < 0$.

The Blue Equation reduces to

$$4\frac{dT_{\rm surface}}{T_{\rm surface}} = \frac{d\tau_S}{1+\tau_S}$$

Again,

$$4\frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{d\tau_S}{1+\tau_S}$$

Suppose the transmissivity decreases from 0.9 to 0.8. Then $\tau_S = 0.9$ and $d\tau_S = -0.1$.

Suppose also that the equilibrium temperature of the Earth with $\tau_S = 0.9$ is 288 K (as we found above).

Then

$$4\frac{dT_{\text{surface}}}{288} = \left(\frac{-0.1}{1.9}\right) \qquad \text{or} \qquad dT_{\text{surface}} = \left(\frac{-0.1}{1.9}\right)\frac{288}{4} = -3.8\,K$$

Thus, the assumed increase in shortwave absorbtivity has resulted in a decrease in surface temperature of about $4°$C.

Sensitivity to Longwave Absorbtion

Suppose next that some change causes an *increase in the absorbtion of terrestrial radiation* by the atmosphere.

For example, a pall of ash in the stratosphere, following a major volcanic eruption, could absorb or scatter a significant proportion of outgoing longwave radiation.

So,

$$a_L \Rightarrow a_L + d\,a_L$$
$$\tau_L \Rightarrow \tau_L + d\,\tau_L$$

Clearly, if $d\,a_L > 0$ then $d\,\tau_L = -d\,a_L < 0$.

The Blue Equation reduces to

$$4\frac{dT_{\text{surface}}}{T_{\text{surface}}} = -\frac{d\tau_L}{1+\tau_L}$$

Again,

$$4\frac{dT_{\text{surface}}}{T_{\text{surface}}} = -\frac{d\tau_L}{1+\tau_L}$$

Suppose the longwave transmissivity decreases from 0.2 to 0.1. Then $\tau_L = 0.2$ and $d\tau_S = -0.1$.

Suppose once more that the equilibrium temperature of the Earth with $\tau_L = 0.2$ is 288 K.

Then

$$4\frac{dT_{\text{surface}}}{288} = -\left(\frac{-0.1}{1.2}\right) \qquad \text{or} \qquad dT_{\text{surface}} = \left(\frac{0.1}{1.2}\right)\frac{288}{4} = 6.0\,K$$

Thus, the assumed increase in longwave absorbtivity has resulted in an increase in surface temperature of about $6°$C.

Sensitivity to Planetary Albedo

Suppose next that some change causes an *increase in the albedo or reflectivity* of the atmosphere.

For example, an increase in condensation nuclei might result in a greater coverage of high-level cirrus cloud, which could reflect a higher proportion of incoming solar radiation.

So,

$$A \Rightarrow A + dA$$

The Blue Equation reduces to

$$4\frac{dT_{\text{surface}}}{T_{\text{surface}}} = -\frac{dA}{1-A}$$

Again,

$$4\frac{dT_{\text{surface}}}{T_{\text{surface}}} = -\frac{dA}{1-A}$$

Suppose the albedo increases from 0.3 to 0.4. Then $A = 0.3$ and $dA = 0.1$.

Suppose once more that the equilibrium temperature of the Earth with $A = 0.3$ is 288 K.

Then

$$4\frac{dT_{\text{surface}}}{288} = -\left(\frac{0.1}{0.7}\right) \qquad \text{or} \qquad dT_{\text{surface}} = -\left(\frac{0.1}{0.7}\right)\frac{288}{4} = -10.3\,K$$

Thus, the assumed increase in albedo has resulted in an decrease in surface temperature of about $10°$C.

Sensitivity to Solar Constant

Suppose next that the *solar energy flux varies*.

This could be the result of a major solar anomaly, or due to a secular or cyclic variation associated, for example, with the sun-spot cycle.

So,

$$F_{\text{solar}} \Rightarrow F_{\text{solar}} + d\,F_{\text{solar}}$$

The Blue Equation reduces to

$$4\frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{dF_{\text{solar}}}{F_{\text{solar}}}$$

Again,

$$4\frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{dF_{\text{solar}}}{F_{\text{solar}}}$$

Suppose the solar output increases by 1%. Then $dF_{\text{solar}}/F_{\text{solar}} = 0.01$.

Suppose once more that the equilibrium temperature of the Earth is 288 K.

Then

$$4\frac{dT_{\text{surface}}}{288} = \left(\frac{0.01\,F_{\text{solar}}}{F_{\text{solar}}}\right) \qquad \text{or} \qquad dT_{\text{surface}} = 0.01 \times \frac{288}{4} = 0.7\,K$$

Thus, the assumed 1% increase in solar flux has resulted in an increase in surface temperature of less than $1°$C.

Review of Sensitivities

- **An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about $4°$C.**
- **An increase in longwave absorbtivity of 0.1 resulted in an increase in surface temperature of about $6°$C.**
- **An increase in albedo of 0.1 resulted in an decrease in surface temperature of about $10°$C.**
- **A 1% increase in solar flux has resulted in an increase in surface temperature of less than $1°$C.**

In general, all parameters undergo small changes. Moreover, we have completely neglected the effects of water vapour and liquid water.

The results give an indication of <u>how difficult it</u> is to gauge the consequences for climate of any changes which may occur.