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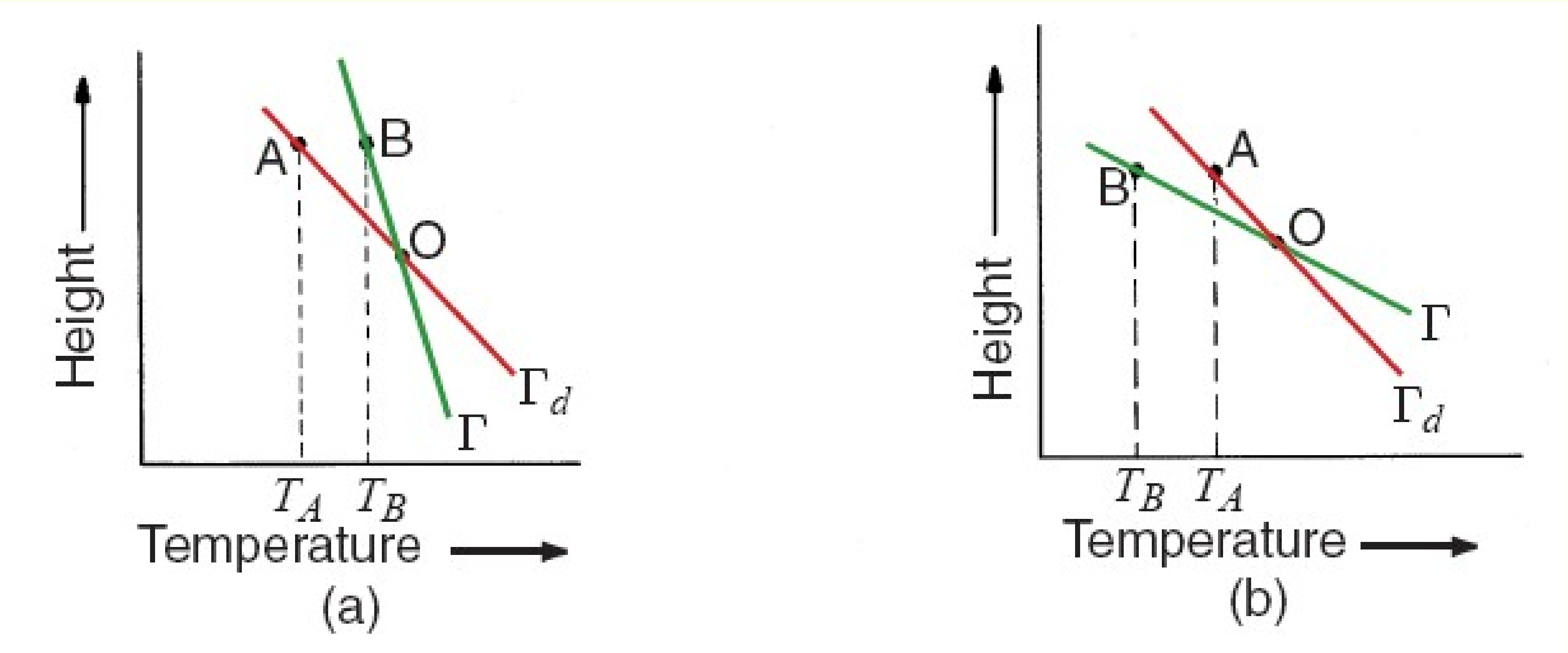
If the parcel is displaced downwards, it becomes warmer than the ambient air and will tend to rise again.

In both cases, the parcel of air encounters a *restoring force* after being displaced, which *inhibits vertical mixing*. Thus, the condition $\Gamma < \Gamma_d$ corresponds to stable stratification (or positive *static stability*) for unsaturated air parcels.



Conditions for (a) positive static stability ($\Gamma < \Gamma_d$) and (b) negative static instability ($\Gamma > \Gamma_d$) for the displacement of unsaturated air.

Exercise: **An unsaturated parcel of air has density ρ' and temperature T', and the density and temperature of the ambient air are ρ and T. Derive an expression for the downward acceleration of the air parcel in terms of T and T'.**

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or, using the gas equation,

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If $(\Gamma_d - \Gamma) < 0$, the solutions are exponentially growing with time. This corresponds to *static instability*.

Exercise: Find the period of oscillation of a parcel of air displaced vertically, where the ambient temperature and lapse-rate are

- $T = 250\,\mathbf{K}$ **and** $\Gamma = 6\,\mathbf{K\,km}^{-1}$**, typical tropospheric values**
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Assuming $\Gamma_d = 10\,\mathbf{K\,km}^{-1} = 0.01\,\mathbf{K\,m}^{-1}$ **and** $g = 10\,\mathbf{m\,s}^{-2}$**,**

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For $\Gamma = -2\,\mathbf{K\,km}^{-1}$**,** ω^2 **is tripled. Thus,** $\tau \approx 290\,\mathbf{s}$**.**

Inversions

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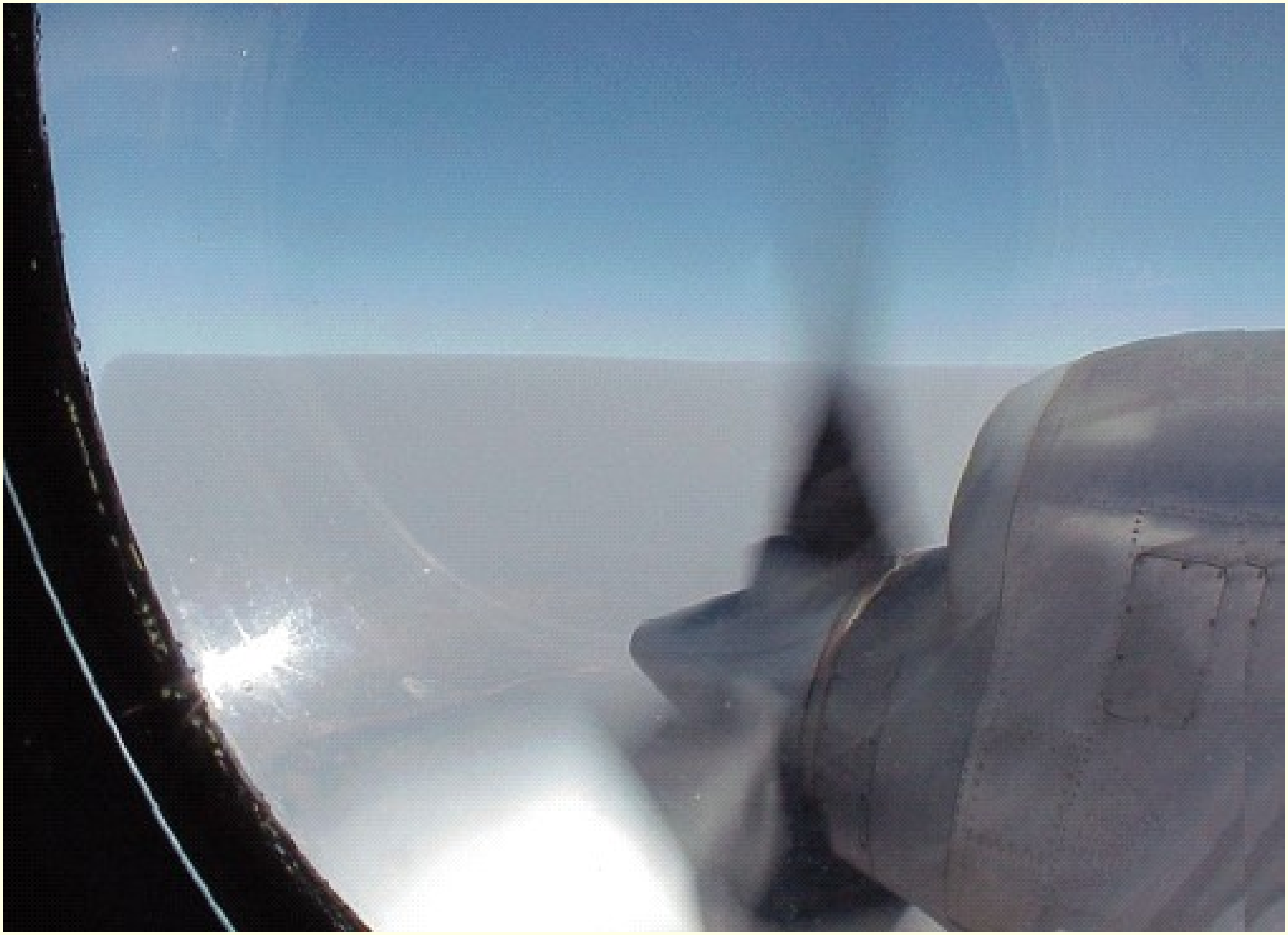
A low-level inversion can act as a *lid* that traps pollution-laden air beneath it (See following figure).

The layered structure of the stratosphere derives from the fact that it represents an inversion in the vertical temperature profile.



Looking down onto widespread haze over southern Africa. The haze is confined below a temperature inversion. Above the inversion, the air is remarkably clean and the visibility is excellent.

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The only exception is in the layer just above the ground under conditions of very strong heating from below.

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Differentiating this yields

$$\begin{aligned} c_p T\frac{d\theta}{\theta} &= c_p\,dT - RT\frac{dp}{p} \\ &= c_p\,dT - \frac{dp}{\rho} \\ &= c_p\,dT + g\,dz\,. \end{aligned}$$

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Thus, *if the potential temperature θ increases with altitude* ($d\theta/dz > 0$) we have $\Gamma < \Gamma_d$ and the atmosphere is *stable* with respect to the displacement of unsaturated air parcels.

Conditional & Convective Instability

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It follows that if Γ is the actual lapse rate, saturated air parcels will be stable, neutral, or unstable with respect to vertical displacements, according to the following scheme:

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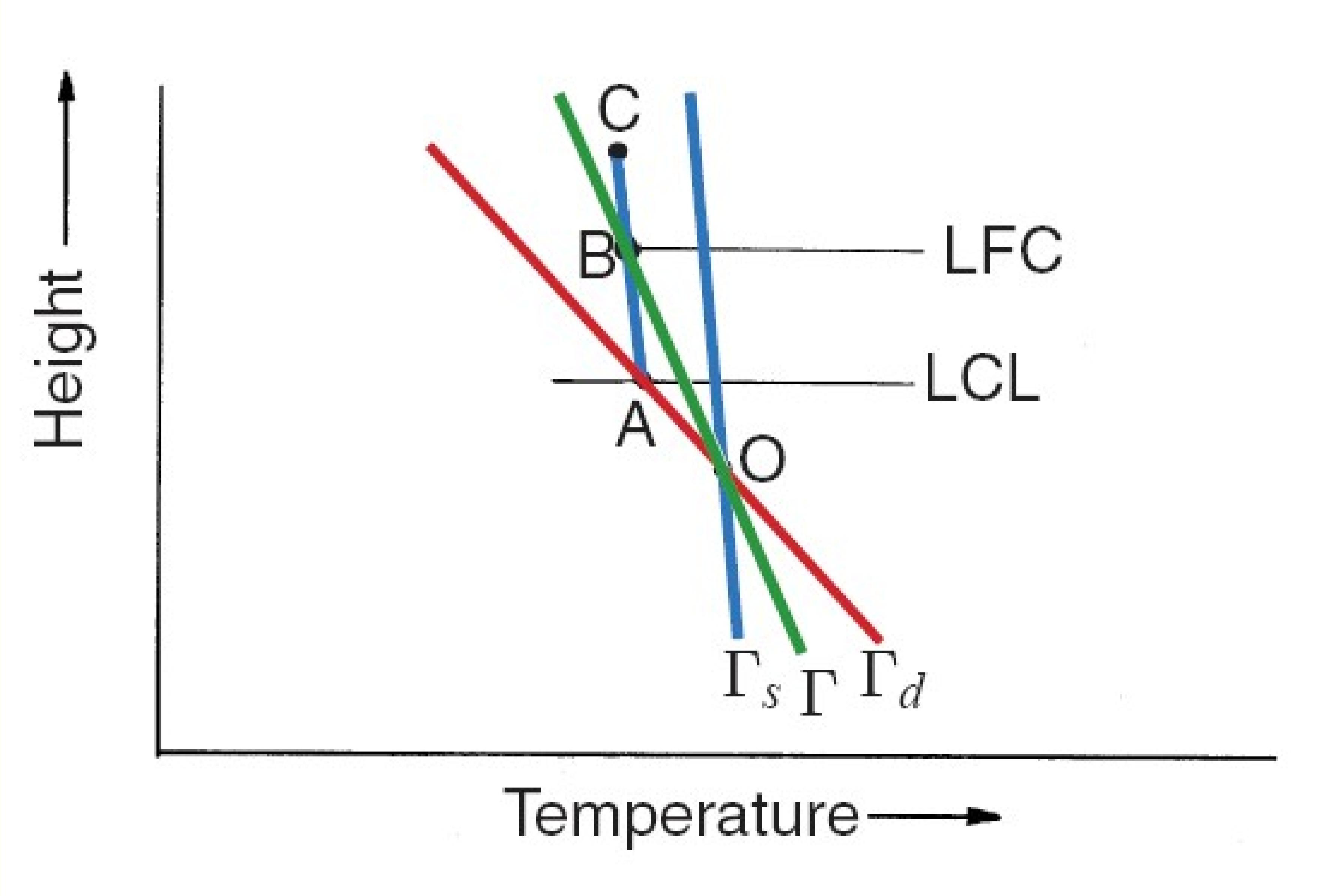
When an environmental temperature sounding is plotted on a tephigram, the distinctions between Γ, Γ_d and Γ_s are clearly discernible.

If the actual lapse rate Γ of the atmosphere lies *between* the saturated adiabatic lapse rate and the dry adiabatic lapse rate,

$$\Gamma_s < \Gamma < \Gamma_d$$

a parcel of air that is lifted *sufficiently far* above its equilibrium level will become warmer than the ambient air. This situation is illustrated in the following figure.



Conditions for conditional instability ($\Gamma_s < \Gamma < \Gamma_d$). LCL is the lifting condensation level and LFC is the *level of free convection*.

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Such an atmosphere is said to be *conditionally unstable* with respect to convection.

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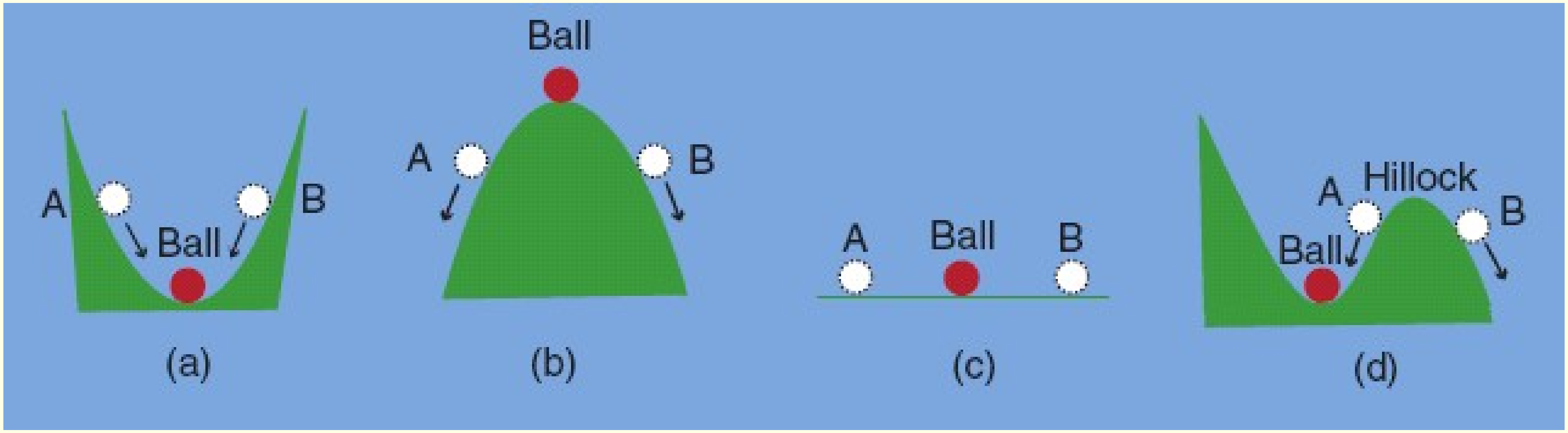

Figure 3.15. Analogs for (a) stable, (b) unstable, (c) neutral, and (d) conditional instability.

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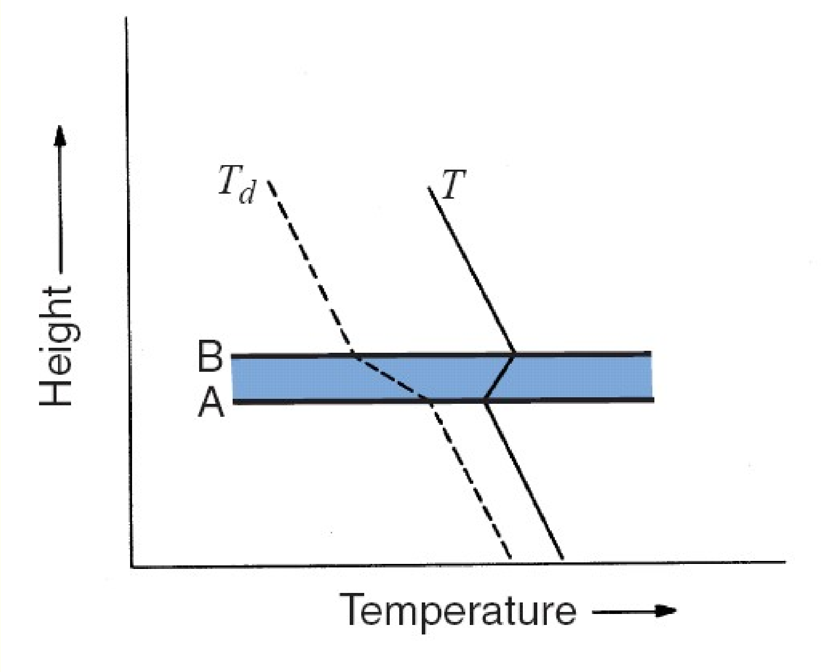

Convective instability. The blue shaded region is a dry inversion layer.

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Therefore, *as the inversion layer is lifted, the top part of it cools much more rapidly than the bottom part*, and the *lapse rate quickly becomes destabilized*.

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Throughout *large areas of the tropics* θ_e decreases markedly with height from the mixed layer to the much drier air above. Yet deep convection breaks out only within a few percent of the area where there is sufficient lifting.

End of §2.6