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We have quantified its effect on the density of air by introducing the concepts of *mixing ratio* and of *virtual temperature*.

However, the amount of water vapour present in a certain quantity of air may be expressed in many different ways, some of the more important of which are considered now.

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Why So Many Moisture Variables?

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- Some are fundamental quantities, related simply to other thermodynamic variables.

  Vapour pressure is hard to measure and not directly useful in applications. But it is theoretically important, describing how saturation humidity varies with temperature.

Mixing Ratio & Specific Humidity

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That is,

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It should be noted that if neither condensation nor evapouration takes place, the mixing ratio of an air parcel is constant (i.e., it is a *conserved quantity*).

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And $p = 1026.8\,$hPa$= 102680\,$Pa.

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And $p = 1026.8\,\mathbf{hPa} = 102680\,\mathbf{Pa}$.

So

$$e = \frac{102680}{0.622} \times 0.0055 = 908\,\mathbf{Pa} \approx 9\,\mathbf{hPa}$$

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Thus

$$T_v - T \approx 0.608\,w\,T = 0.608 \times 0.02 \times 303 = 3.68\,\mathbf{K}$$

Therefore, the virtual temperature elevation is 3.68°C.

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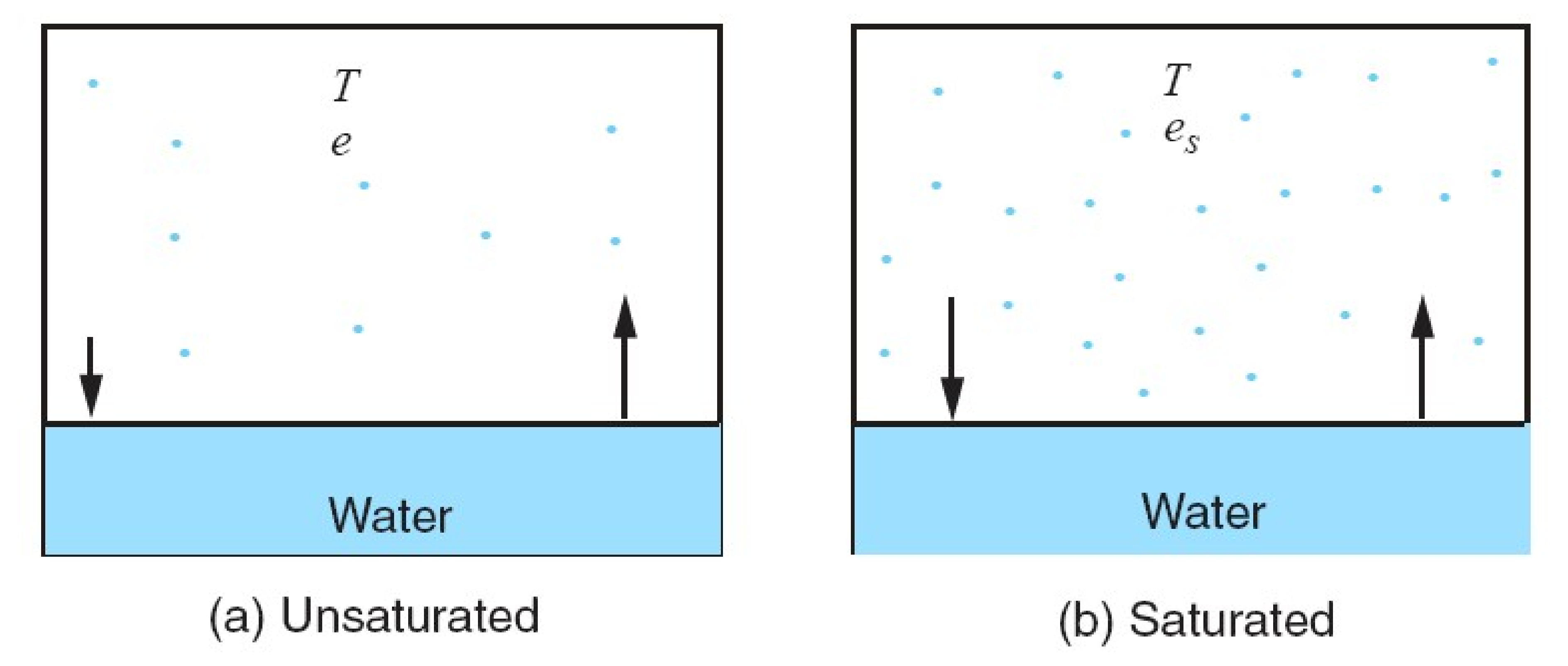
If the rate of condensation is less than the rate of evapouration, the box is said to be *unsaturated*.

When the water vapour pressure in the box increases to the point that the rate of condensation is equal to the rate of evapouration, the air is said to be *saturated*.



(a) Unsaturated

(b) Saturated

Figure 3.8. A box (a) unsaturated and (b) saturated with respect to a plane surface of pure water at temperature T. The vapour pressure over a plane surface of pure water at temperature T is e_s.

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Similarly, if the water were replaced by a plane surface of *pure ice* at temperature T, and the rate of condensation of water vapour were equal to the rate of evapouration of the ice, the pressure e_{si} exerted by the water vapour would be the *saturation vapour pressure over a plane surface of pure ice* at temperature T.

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Since, at any given temperature, the rate of evapouration from ice is less than from water, $e_{si}(T) < e_s(T)$.

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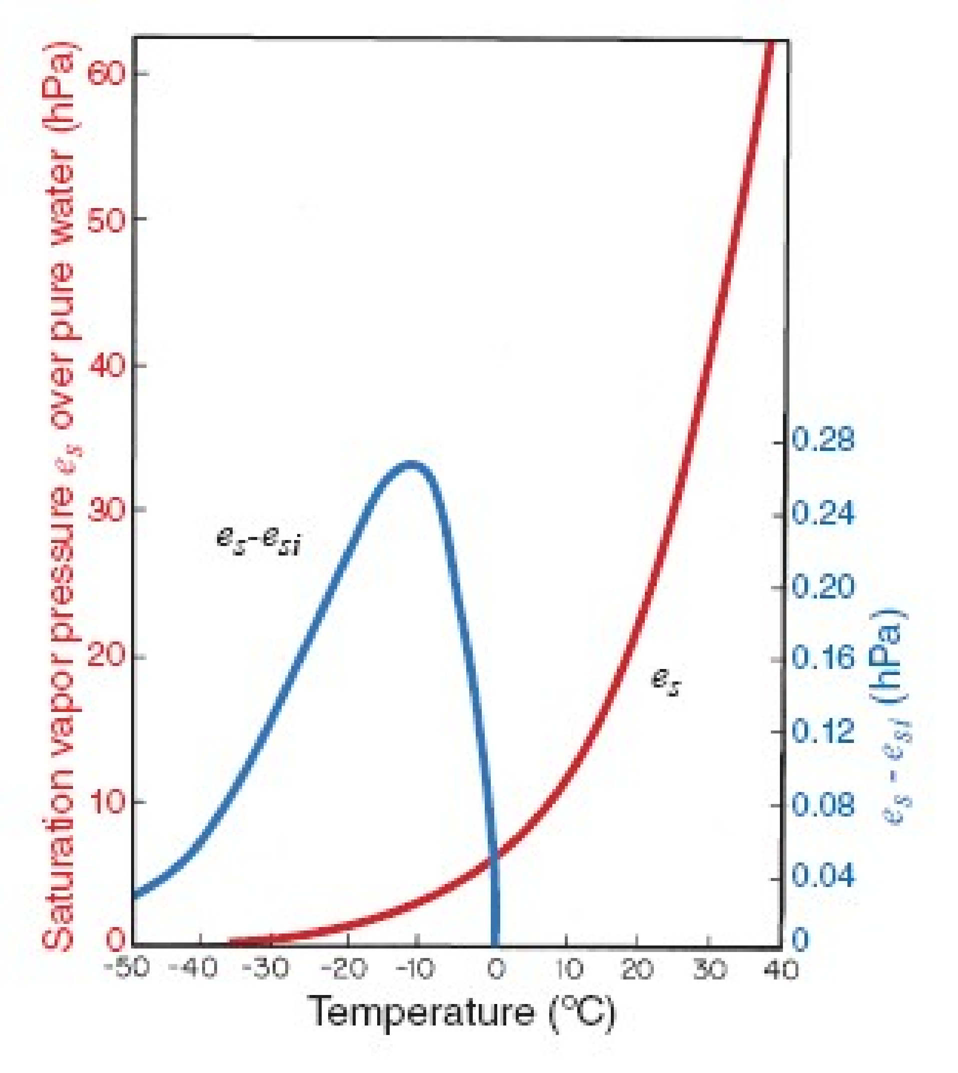
We will see later that this phenomenon plays a role in the growth of precipitable particles in some clouds.



Variations with temperature of the saturation vapour pressure e_s over a plane surface of pure water (red line). Difference $e_s - e_{si}$ between saturation vapour pressures over water and ice (blue line).

Saturation Mixing Ratio

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Since water vapour and dry air both obey the ideal gas equation,

$$w_s = \frac{\rho_s}{\rho_d} = \frac{e_s/R_v T}{(p - e_s)/R_d T} = \frac{R_d}{R_v} \frac{e_s}{p - e_s}$$

where ρ_s is the partial density of water vapour required to saturate air with respect to water at temperature T, ρ_d is the partial density of the dry air, and p is the total pressure.

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Hence, at a given temperature, the saturation mixing ratio is *inversely proportional to the total pressure.*

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Lines of constant saturation mixing ratio are printed as *dashed lines on the tephigram* and are labeled with the value of w_s in grams of water vapour per kilogram of dry air.

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Exercise: **Check the above statement (1) by examination of the tephigram and (2) by analytical means (requiring the Clausius-Clapeyron Equation).**

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In other words, the dew point is the temperature at which the saturation mixing ratio w_s with respect to liquid water becomes equal to the actual mixing ratio w.

It follows that the humidity at temperature T and pressure p is given by

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Exercise: **Air at 1000 hPa and 18°C has a mixing ratio of $6\,\mathrm{g\,kg^{-1}}$. What are the relative humidity and dew point of the air?**

Solution: **46%, 6.5°C. This exercise may be solved using the tephigram chart.**

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Fortunately, dew points much above this temperature are rarely observed even in the tropics.

Thermal Comfort

At the Earth's surface, the pressure varies only slightly from place to place and from time to time. Therefore, the dew point is a good indicator of the moisture content of the air.

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Fortunately, dew points much above this temperature are rarely observed even in the tropics.

In contrast to the dew point, relative humidity depends as much upon the temperature of the air as upon its moisture content.

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The highest dew points occur over warm bodies of water or vegetated surfaces from which water is evapourating.

In the absence of vertical mixing, the air just above these surfaces would become saturated with water vapour, at which point the dew point would be the same as the temperature of the underlying surface.

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Complete saturation is rarely achieved over hot surfaces, but dew points in excess of 25°C are sometimes observed over the warmest regions of the oceans.

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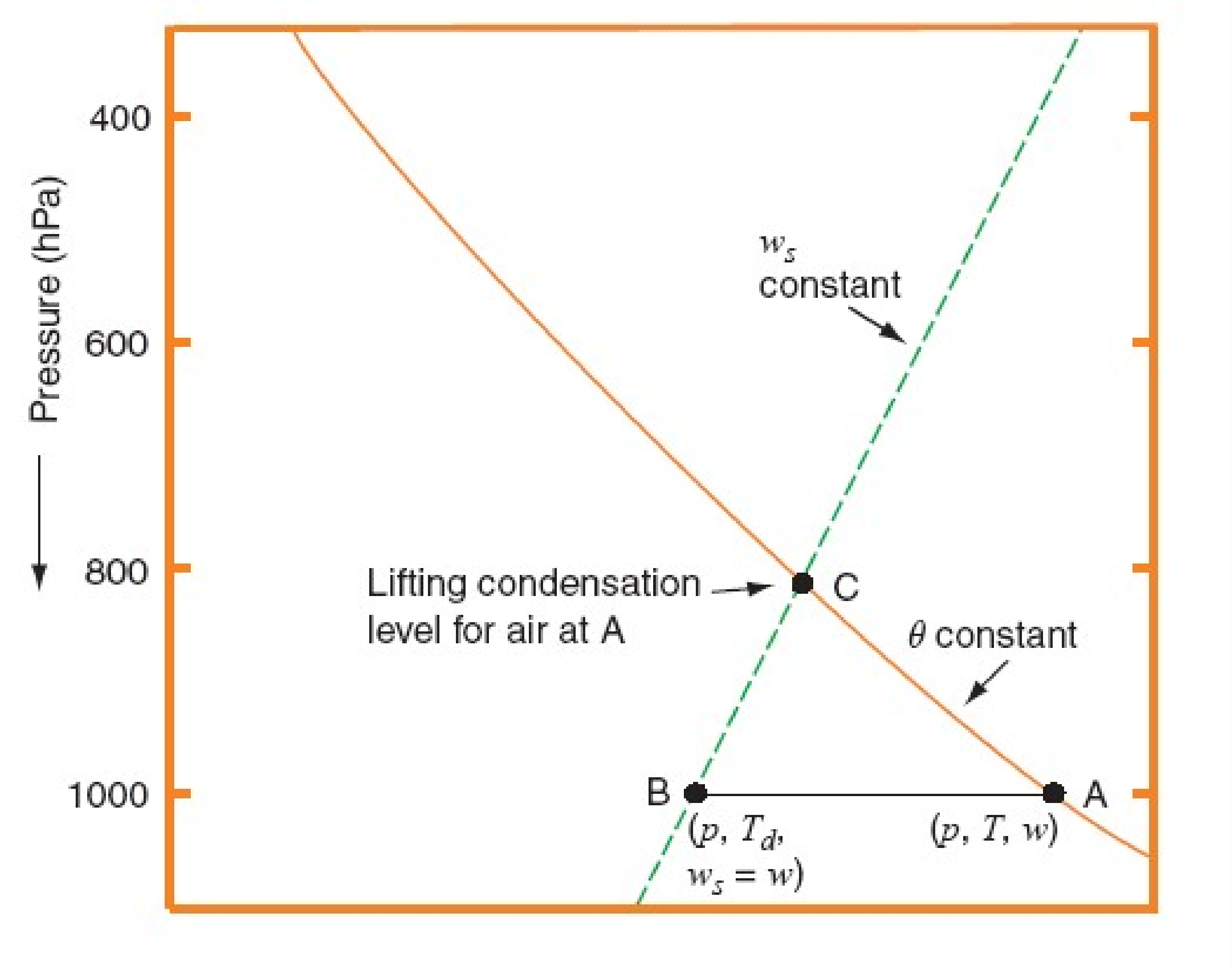
Therefore, the LCL is located at the intersection of the potential temperature line passing through the temperature T and pressure p of the parcel of air, and the w_s line that passes through the pressure p and dew point T_d of the air parcel (see figure).



The lifting condensation level of a parcel of air at A, with pressure p, temperature T and dew point T_d, is at point C.

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Similarly, a knowledge of pressure, temperature and *any one moisture parameter* is sufficient to determine all the other moisture parameters we have defined.

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If a raindrop falls through a layer of air that has a constant wet-bulb temperature, the raindrop will eventually reach a temperature equal to the wet-bulb temperature of the air.

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If the air approaching the wet bulb is unsaturated, w' is greater than w ; therefore, $T_d \leq T_w \leq T$.

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The temperature at which this phase change occurs is called the *melting point*.

At 1000 hPa and $0°$C, the latent heat of melting of the water substance is $3.34 \times 10^5\,\mathbf{J\,kg}^{-1}$. Note that:

$$\begin{bmatrix}\textbf{latent heat}\\\textbf{of melting}\end{bmatrix} = \begin{bmatrix}\textbf{latent heat}\\\textbf{of fusion}\end{bmatrix} = \begin{bmatrix}\textbf{latent heat}\\\textbf{of freezing}\end{bmatrix}.$$

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For water at 1000 hPa and 100°C the latent heat of vapourization is $2.25 \times 10^6\,\mathbf{J\,kg^{-1}}$.

$$\begin{bmatrix}\textbf{latent heat}\\ \textbf{of vaporization}\end{bmatrix} = \begin{bmatrix}\textbf{latent heat}\\ \textbf{of condensation}\end{bmatrix} = \begin{bmatrix}\textbf{latent heat}\\ \textbf{of boiling}\end{bmatrix}.$$

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If all of the condensation products remain in the rising parcel, the process may still be considered to be adiabatic (and *reversible*), even though latent heat is released in the system.

The air parcel is then said to undergo a *saturated adiabatic process*.

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However, the amount of heat carried by condensation products is small compared to that carried by the air itself.

Therefore, the saturated-adiabatic lapse rate is essentially the same as the pseudoadiabatic lapse rate.

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$$\frac{dT}{dz} = -\frac{L_v}{c_p}\left(\frac{\partial w_s}{\partial T}\right)_p \frac{dT}{dz} - \frac{g}{c_p}$$

Rearranging the last expression yields

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where Γ_s is called the saturated adiabatic lapse rate, which is the rate of decrease in temperature with height of an air parcel undergoing a saturated adiabatic process.

The magnitude of Γ_s is not constant but depends on the pressure and temperature. Since $(\partial w_s/\partial T)_p$ is always positive, it follows that

$$\Gamma_s < \Gamma_d$$

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Lines that show the decrease in temperature with height of a parcel of air that is rising or sinking in the atmosphere under saturated adiabatic (or pseudoadiabatic) conditions are called *saturated adiabats* (or pseudoadiabats).

On the tephigram, these are the curved lines that diverge upward and tend to become parallel to the dry adiabats at lower pressure values.

Exercise: **A parcel of air with an initial temperature of $15°$C and dew point $2°$C is lifted adiabatically from the 1000 hPa level. Determine its LCL and temperature at that level. If the air parcel is lifted a further 200 hPa above its LCL, what is its final temperature and how much liquid water is condensed during this rise?**

Exercise: **A parcel of air with an initial temperature of 15°C and dew point 2°C is lifted adiabatically from the 1000 hPa level. Determine its LCL and temperature at that level. If the air parcel is lifted a further 200 hPa above its LCL, what is its final temperature and how much liquid water is condensed during this rise?**

Solution: **Solve using a tephigram chart.**

William Thomson (Lord Kelvin) was the first (in 1862) to derive quantitative estimates of the dry and saturated adiabatic lapse rates based on theoretical arguments.

Equivalent Potential Temperature

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Combining the gas law ($p\alpha = RT$) and the thermodynamic equation ($dq = c_p dT - \alpha dp$) we have

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The potential temperature θ **is given by**

$$\theta = T \left(\frac{p}{p_0} \right)^{-R/c_p}$$

or, taking logarithms of both sides,

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Differentiating this equation gives

$$c_p\frac{d\theta}{\theta} = \left\{ c_p\frac{dT}{T} - R\frac{p}{p} \right\}$$

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"It can be shown that"

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This last expression can be integrated to give

$$-\left(\frac{L_v w_s}{c_p T}\right) \approx \log\theta + \mathbf{const}$$

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It can be seen that θ_e is the potential temperature θ of a parcel of air when its saturation mixing ratio w_s is zero.

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(If the air is initially unsaturated, w_s and T are the saturation mixing ratio and temperature at the point where the air first becomes saturated after being lifted dry adiabatically.)

The equivalent potential temperture is conserved during both dry and saturated adiabatic processes.

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If the line of constant equivalent potential temperature (that is, the pseudoadiabat) that passes through the wet-bulb temperature of a parcel of air is traced back on a tephigram to the point where it intersects the 1000 hPa isobar, the temperature at this intersection is called the *wet-bulb potential temperature* θ_w of the air parcel.

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Like the equivalent potential temperature, the wet-bulb potential temperature is *conserved during both dry and saturated adiabatic processes*. Both θ_w and θ_e are valuable as *tracers* of air masses.

Wet-bulb Potential Temperature

If the line of constant equivalent potential temperature (that is, the pseudoadiabat) that passes through the wet-bulb temperature of a parcel of air is traced back on a tephigram to the point where it intersects the 1000 hPa isobar, the temperature at this intersection is called the *wet-bulb potential temperature* θ_w of the air parcel.

Like the equivalent potential temperature, the wet-bulb potential temperature is *conserved during both dry and saturated adiabatic processes*. Both θ_w and θ_e are valuable as *tracers* of air masses.

When height, rather than pressure, is used as the independent variable, the conserved quantity during adiabatic or pseudoadiabatic ascent or descent with water undergoing transitions between liquid and vapour phases is the *moist static energy*.

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The *moist static energy* (MSE) is defined as

$$MSE = c_p T + \Phi + L_v q$$

where T is the temperature of the air parcel, Φ is the geopotential and q the specific humidity (recall $q \approx w$).

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- **The first term ($c_p T$) is the enthalpy per unit mass of air**
- **The second term (Φ) is the potential energy**
- **The third term ($L_v q$) the latent heat content.**

The first two terms comprise the *dry static energy*.

Recall the definition of moist static energy:

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When air is lifted dry adiabatically, enthalpy is converted into potential energy and the latent heat content remains unchanged.

In saturated adiabatic ascent, energy is exchanged among all three terms: potential energy increases, while enthalpy and latent heat content both decrease. However, the sum of the three terms remains constant.

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- **The saturation mixing ratio line that passes through the point determined by the dew point and pressure of the air.**

This rule is illustrated in the following figure.

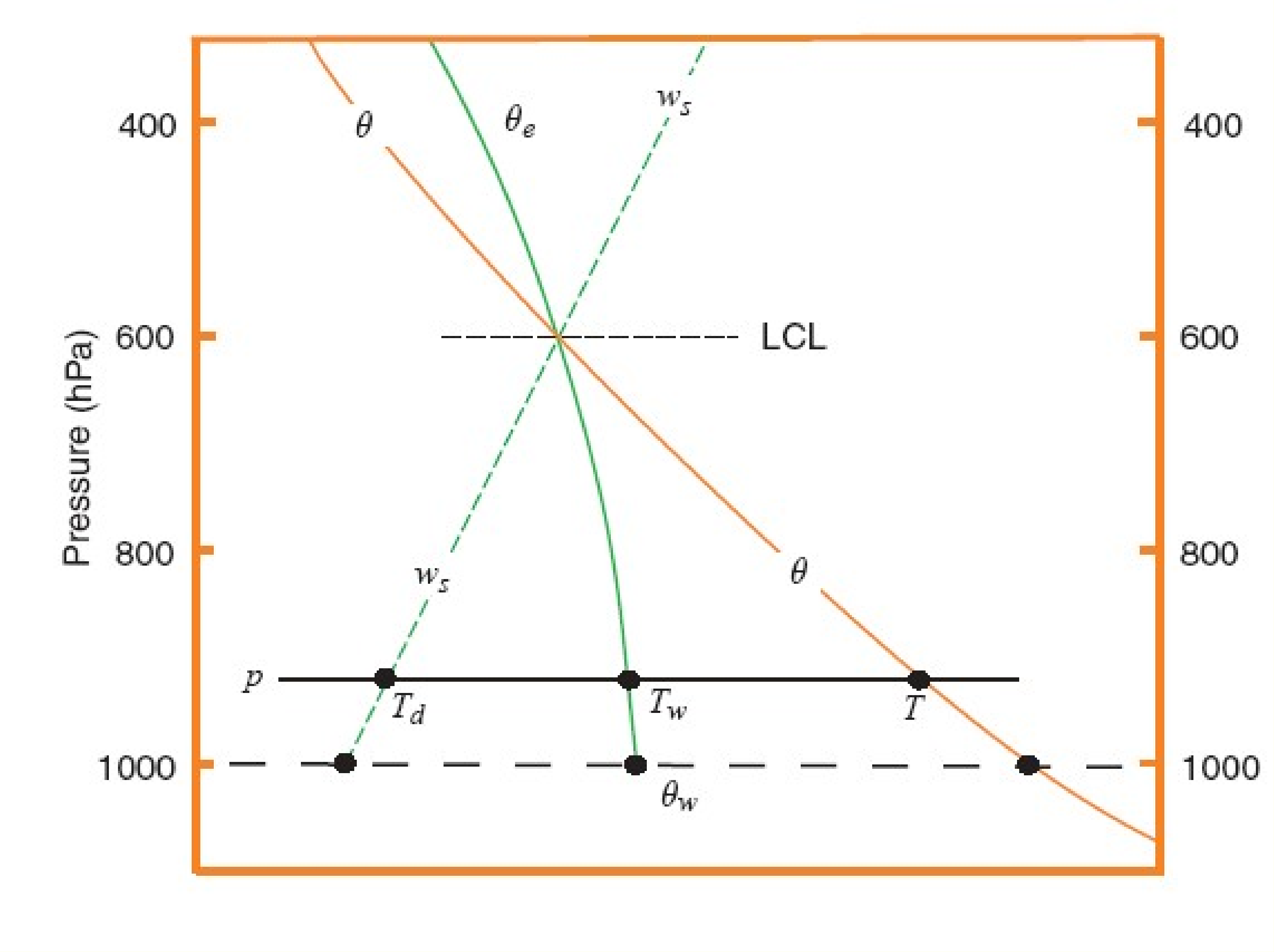

Figure 3.11. Illustration of Normand’s rule.

Consider an air parcel with temperature T, pressure p, dew point T_d, and wet-bulb temperature T_w.

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Also, by extrapolating the θ_e line that passes through T_w to the 1000 hPa level, the wet-bulb potential temperature θ_w, may be found (see figure).

Ascent Followed by Descent

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If a parcel of air is lifted above its LCL so that condensation occurs, and if the products of the condensation fall out as precipitation, the latent heat gained by the air during this process will be retained by the air if the parcel returns to its original level.

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The effects of the saturated ascent coupled with the adiabatic descent are:

- **Net increases in the temperature and potential temperature of the parcel**
- **A decrease in moisture content (as indicated by changes in the mixing ratio, relative humidity, dew point and wet-bulb temperature)**
- **No change in the equivalent potential temperature or wet-bulb potential temperature, which are conserved quantities for air parcels undergoing both dry and saturated processes.**

Exercise

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The following exercise illustrates the effects of ascent and descent.

(1) An air parcel at 950 hPa has a temperature of 14°C and a mixing ratio of $8\,\mathrm{g\,kg^{-1}}$. What is the wet-bulb potential temperature of the air?

(2) The air parcel is lifted to the 700 hPa level by passing over a mountain, and 70% of the water vapour that is condensed out by the ascent is removed by precipitation. Determine the temperature, potential temperature, mixing ratio, and wet-bulb potential temperature of the air parcel after it has returned to the 950 hPa level on the other side of the mountain.

Solution

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On a tephigram, locate the initial state of the air at 950 hPa and $14°\text{C}$. The saturation mixing ratio for an air parcel with temperature and pressure is found from the chart to be $10.6\,\text{g}\,\text{kg}^{-1}$.

Therefore, since the air has a mixing ratio of only $8\,\text{g}\,\text{kg}^{-1}$, it is unsaturated. The wet-bulb potential temperature can be determined using the following method: Trace the constant potential temperature line that passes through the initial state of the air parcel up to the point where it intersects the saturation mixing ratio line with value $8\,\text{g}\,\text{kg}^{-1}$. This occurs at a pressure of about 890 hPa, which is the LCL of the air parcel. Now follow the equivalent potential temperature line that passes through this point back down to the 1000 hPa level and read off the temperature on the abscissa — it is $14°\text{C}$. This is in the wet-bulb potential temperature θ_w of the air.

When the air is lifted over the mountain, its temperature and pressure up to the LCL at 890 hPa are given by points on the potential temperature line that passes through the point 950 hPa and $14°\text{C}$. With further ascent of the air parcel to the 700 hPa level, the air follows the saturated adiabat that passes through the LCL. This saturated adiabat intersects the 700 hPa level at a point where the saturation mixing

ratio is $4.7\,\mathrm{g\,kg^{-1}}$. Therefore, $8 - 4.7 = 3.3\,\mathrm{g\,kg^{-1}}$ of water vapour has to condense out between the LCL and the 700 hPa level, and 70% of this, or $2.3\,\mathrm{g\,kg^{-1}}$, is precipitated out. Therefore, at the 700 hPa level there is $1\,\mathrm{g\,kg^{-1}}$ of liquid water in the air. The air parcel descends on the other side of the mountain at the saturated adiabatic lapse rate until it evapourates all of its liquid water, at which point the saturation mixing ratio will have risen to $4.7 + 1 = 5.7\,\mathrm{g\,kg^{-1}}$. The air parcel is now at a pressure of 760 hPa and a temperature of 1.8°C. Thereafter, the air parcel descends along a dry adiabat to the 950 hPa level, where its temperature is 20°C and the mixing ratio is still $5.7\,\mathrm{g\,kg^{-1}}$. If the method indicated in the Figure above is applied again, the wet-bulb potential temperature of the air parcel will be found to be unchanged at 14°C.

Mountain Winds

The heating of air during its passage over a mountain, 6°C in the above example, is responsible for the remarkable warmth of the *Föhn* and *Chinook* winds, which often blow downward along the lee side of mountain ranges.

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Aside: **The person who first explained the Föhn wind in this way appears to have been Julius F. von Hann in his classic book *Lehrbuch der Meteorologie* (Leipzig, 1901). J. von Hann (1839-1921) was an Austrian meteorologist, who introduced thermodynamic principles into meteorology. He developed theories for mountain and valley winds. He also published the first comprehensive treatise on climatology (1883).**

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Comment: **Remark on mean winds in Rio and Buenos Aires.**

End of §2.5