

CHAPTER 4: SOME OTHER FUNCTIONS

1. THE ABSOLUTE VALUE

1.1. The definition and graph of the absolute value.

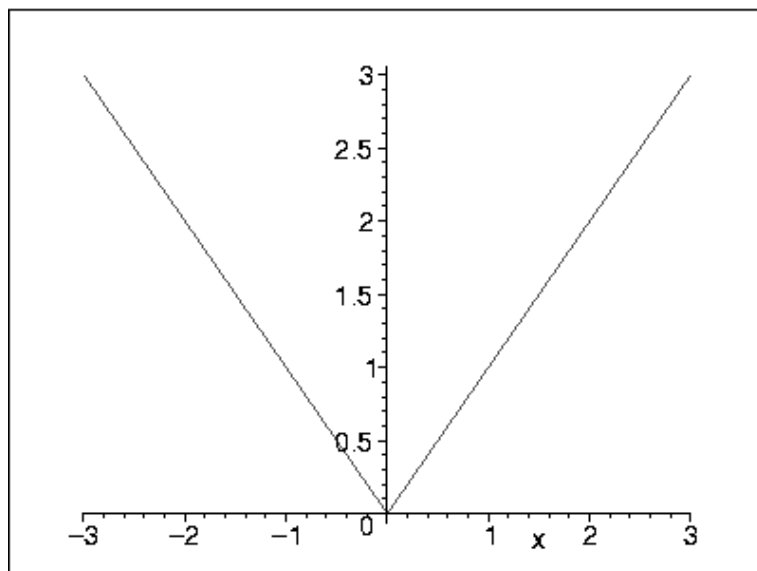
Definition 1.1. For a real number a , the *absolute value of a* , denoted $|a|$, is the distance from a to 0 on the real line; i.e.,

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Example 1.1. $|5| = 5$, $|3.689| = 3.689$,
 $|-5| = 5$, $|-3.689| = 3.689$, $|\pi| = \pi$, $|0| = 0 \dots$ etc.

For $x > 0$, $|x| = x$ and thus, for $x > 0$ (that is, to the right of the y -axis) the graph of $f(x) = |x|$ is just the line $y = x$. Similarly, to the left of the y -axis the graph of $f(x) = |x|$ is the line $y = -x$ (with slope -1).

The graph of $f(x) = |x|$:



Note that if $a, b \in \mathbb{R}$ then $|a - b|$ = the distance from a to b on the real line.

Example 1.2. $|2 - 5| = |-3| = 3$ = the distance from 2 to 5. $|4 - (-3)| = |4 + 3| = |7| = 7$ = the distance from 4 to -3 .

1.2. Inequalities involving absolute values. Note: If $a, b \in \mathbb{R}$ with $b > 0$ then $|a| < b \iff$ the distance from a to 0 is less than $b \iff a$ lies between $-b$ and b on the real line $\iff -b < a < b$.

Similarly $|a| \leq b \iff -b \leq a \leq b$

Example 1.3. Solve $|x - 3| < 4$.

Solution:

$$\begin{aligned} -4 &< x - 3 < 4 \\ -1 &< x < 7 \end{aligned}$$

Answer: $(-1, 7)$

Example 1.4. Solve $|3x - 5| \leq 2$

Solution:

$$\begin{aligned} -2 &\leq 3x - 5 \leq 2 \\ 3 &\leq 3x \leq 7 \\ 1 &\leq x \leq 7/3 \end{aligned}$$

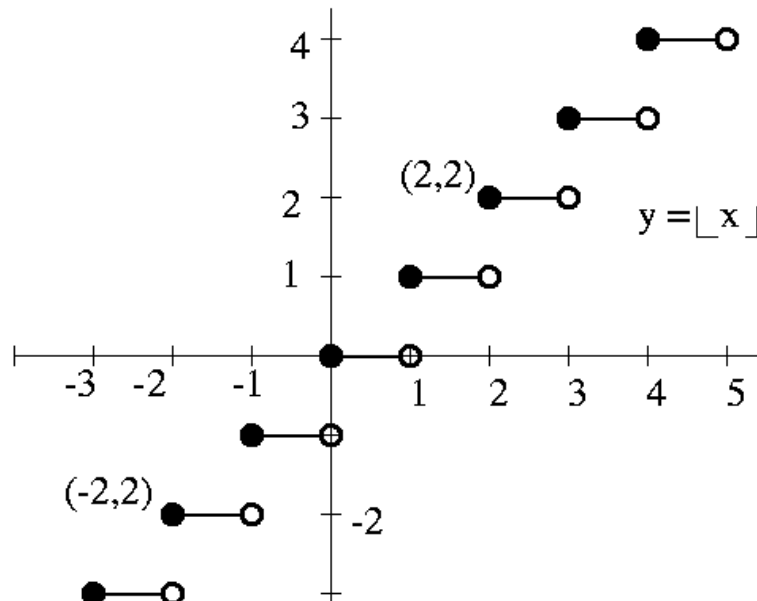
Answer: $[1, 7/3]$

2. SOME FUNCTIONS WITH DISCRETE VALUES

Definition 2.1. (The ‘greatest integer function’): For $a \in \mathbb{R}$, $\lfloor a \rfloor$ denotes the largest integer which is less than or equal to a .

Example 2.1. $\lfloor 1.5 \rfloor = 1$, $\lfloor 3.782 \rfloor = 3$, $\lfloor -1.5 \rfloor = -2$, $\lfloor 7 \rfloor = 7$, $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4 \dots$ etc.

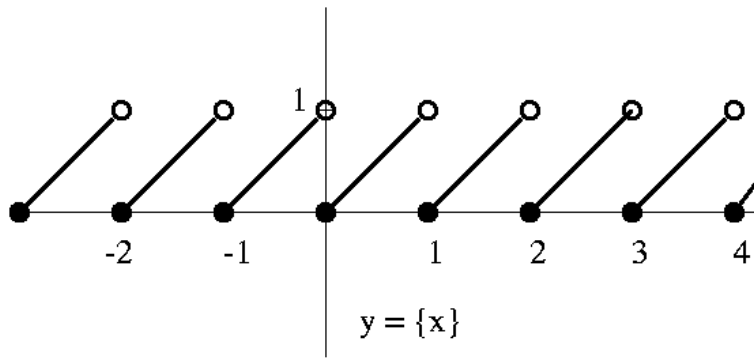
The graph of $f(x) = \lfloor x \rfloor$ (“digitized line”)



Definition 2.2. (The ‘fractional part’):

For $a \in \mathbb{R}$ the fractional part of a , denoted $\{a\}$, is $a - \lfloor a \rfloor$.

Example 2.2. $\{3.782\} = 0.782$, $\{1.5\} = 0.5$, $\{587.312\} = 0.312 \dots$ etc.



“Digitized Wave Function” : $f(x) = (-1)^{\lfloor x \rfloor}$

