MATH1200 2004/5

Homework 21 (Calculus)

Solutions

1. (i) [Recall that $\int e^{ax+b}\,dx = \int \exp(ax+b)\,dx$

$= \frac{1}{a}e^{ax+b} + C = \frac{1}{a}\exp(ax+b) + C$]

$$\int e^{5x}\,dx = \boxed{\frac{1}{5}e^{5x} + C.}$$

(ii) $$\int \exp(2x-8)\,dx = \boxed{\frac{1}{2}\exp(2x-8) + C}$$

(iii) $$\int x^4 e^{x^5}\,dx = \int x^4 \exp(x^5)\,dx.$$

Substitute $u = x^5$. $du = 5x^4\,dx$ So $\frac{1}{5}du = x^4\,dx$.

$$= \frac{1}{5}\int \exp(u)\,du = \frac{1}{5}\exp(u) + C = \boxed{\frac{1}{5}\exp(x^5) + C}$$

(iv) $\int x\exp(x^2+1)\,dx$. Let $u = x^2+1$: $\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int \exp(u)\,du = \frac{1}{2}\exp(u) + C = \boxed{\frac{1}{2}\exp(x^2+1) + C}$$

(v) $\int_1^2 x^2\exp(x^3)\,dx$. Let $u = x^3$: $\frac{1}{3}du = x^2\,dx$

$$\int x^2\exp(x^3)\,dx = \frac{1}{3}\int \exp(u)\,du = \frac{1}{3}\exp(x^3) + C$$

So $$\int_1^2 x^2\exp(x^3)\,dx = \frac{1}{3}\exp(x^3)\Big|_1^2 = \frac{1}{3}\exp(8) - \frac{1}{3}\exp(1)$$

$$= \boxed{\frac{1}{3}(e^8 - e)}$$

(vi) $$\int 10^x\,dx = \int e^{\ln 10\, x}\,dx = \frac{1}{\ln 10}e^{\ln 10\, x} + C = \boxed{\frac{10^x}{\ln 10} + C}$$

2 $\int \frac{\ln x}{x}\,dx = \int \ln x \cdot \frac{1}{x}\,dx.$

Substitution: Let $u = \ln x.$ $du = \frac{1}{x}dx.$

$$= \int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(\ln x)^2 + C.}$$

3 (i) $\int x e^{3x} dx.$ Parts: $\left(\begin{array}{ll} u = x & dv = e^{3x}dx \\ du = dx & v = \frac{1}{3}e^{3x}dx \end{array}\right)$

$$= uv - \int v\,du = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx$$

$$= \boxed{\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C}$$

(ii) $\int x\sin(11x)\,dx.$ $\left(\begin{array}{lll} \text{Parts:} & u = x & dv = \sin(11x)dx \\ & du = dx & v = -\frac{1}{11}\cos(11x) \end{array}\right)$

$$= -\frac{1}{11}x\cos(11x) + \frac{1}{11}\int \cos(11x)\,dx$$

$$= \boxed{-\frac{x}{11}\cos(11x) + \frac{1}{121}\sin(11x) + C}$$

(iii) $\int 4x\cos(-2x)\,dx$ $\left(\begin{array}{lll} \text{Parts:} & u = 4x & dv = \cos(-2x)dx \\ & du = 4\,dx & v = -\frac{1}{2}\sin(-2x) \end{array}\right)$

$$= -2x\sin(-2x) + 2\int \sin(-2x)\,dx$$

$$= -2x\sin(-2x) + 2\cdot\left(\frac{1}{2}\cos(-2x)\right) + C$$

$$= \boxed{-2x\sin(-2x) + \cos(-2x) + C}$$

(iv) $\int x\ln x\,dx$ $\left(\begin{array}{lll} \text{Parts:} & u = \ln x & dv = x\,dx \\ & du = \frac{1}{x}dx & v = \frac{x^2}{2} \end{array}\right)$

$$= \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \boxed{\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C}$$