

Homework 9 (Calculus): Solutions

1. (i) $\frac{d}{dx}(5-8x^3) = \boxed{-24x^2}$ (ii) $\frac{d}{dx}(3+7x-11x^2+15x^3-6x^4) = 7-22x+45x^2-24x^3$

(iii) $\frac{d}{dx}(8x^{10}-11x^8) = 88x^9-88x^7$

2 (i) $\frac{d}{dt}\left(\frac{t^2+3t+2}{t^4+t^2+1}\right) = \frac{(t^4+t^2+1) \cdot (2t+3) - (t^2+3t+2) \cdot (4t^3+2t)}{(t^4+t^2+1)^2}$

$= \frac{2t^5+2t^3+2t+3t^4+3t^2+3-4t^5-2t^3-12t^4-6t^2-6t^3-4t}{(t^4+t^2+1)^2}$

$= \boxed{\frac{t^5-9t^4-6t^3-3t^2-2t+3}{(t^4+t^2+1)^2}}$

(ii) $\frac{d}{dt}\left(\frac{1}{t^2+1}\right) = \frac{(t^2+1) \cdot 0 - 1 \cdot 2t}{(t^2+1)^2} = \boxed{-\frac{2t}{(t^2+1)^2}}$

(iii) $\frac{d}{dt}\left(\frac{t+1}{t^2+1}\right) = \frac{(t^2+1) \cdot 1 - (t+1) \cdot 2t}{(t^2+1)^2} = \frac{t^2+1-2t^2-2t}{(t^2+1)^2} = \boxed{-\frac{t^2-2t+1}{(t^2+1)^2}}$

3. It strikes the ground when $s = 4.9t^2 = 1000 \Rightarrow t = \sqrt{\frac{1000}{4.9}} = 14.3$ secs.

Its speed at time t is $\frac{ds}{dt} = 9.8t$.

\therefore Its speed at time of impact is $9.8 \cdot (14.3) = \boxed{140 \text{ m/s.}}$

4. Speed is $v = \frac{ds}{dt} = 25t - 5t^2$

$v=20 \Leftrightarrow 25t - 5t^2 = 20 \Leftrightarrow 5t^2 - 25t + 20 = 0$

$\Leftrightarrow 5(t^2 - 5t + 4) = 0$

$\Leftrightarrow t = 1$ or $t = 4$

The speed is 20 km/hr at time $t = 1$ hr and $t = 4$ hours.

$v=0$ when $25t - 5t^2 = 0$: when $\boxed{t = 5}$

The car turns around after 5 hours.

5. (i) $\frac{d}{dx}(\sqrt{x^4+1}) = \frac{1}{2\sqrt{x^4+1}} \cdot 4x^3 = \frac{2x^3}{\sqrt{x^4+1}}$

$$5 \text{ (ii)} \quad \frac{d}{dx} \left((\sqrt{x}+2)^{11} \right) = 11 (\sqrt{x}+2)^{10} \cdot \frac{1}{2\sqrt{x}}$$

$$\text{(iii)} \quad \frac{d}{dx} \left((2x^2+5x+1)^{1/3} \right) = \frac{1}{3} (2x^2+5x+1)^{-2/3} \cdot (4x+5)$$

$$\text{(iv)} \quad \frac{d}{dx} \left((x^5-x)^{-1/2} \right) = -\frac{1}{2} (x^5-x)^{-3/2} \cdot (5x^4-1)$$

$$6. \text{ (i)} \quad \frac{d}{dx} \left(x^2 \sqrt{x^2+5} \right) = x^2 \cdot \frac{1}{2\sqrt{x^2+5}} \cdot 2x + 2x\sqrt{x^2+5}$$

$$= \frac{x^3}{\sqrt{x^2+5}} + 2x\sqrt{x^2+5} = \frac{x^3 + 2x(x^2+5)}{\sqrt{x^2+5}}$$

$$= \boxed{\frac{3x^3 + 10x}{\sqrt{x^2+5}}}$$

$$\text{(ii)} \quad \frac{d}{dx} \left(\frac{5x+4}{\sqrt{x^3+1}} \right) = \frac{\sqrt{x^3+1} \cdot 5 - (5x+4) \cdot \frac{1}{2\sqrt{x^3+1}} \cdot 3x^2}{(x^3+1)}$$

$$= \frac{2 \cdot (x^3+1) \cdot 5 - (5x+4) \cdot 3x^2}{2(x^3+1)\sqrt{x^3+1}} = \frac{10x^3+10-15x^3-12x^2}{2(x^3+1)^{3/2}}$$

$$= \boxed{\frac{-5x^3-12x^2+10}{2(x^3+1)^{3/2}}}$$

$$7. \text{ (i)} \quad y = x^{1/3} : \frac{dy}{dx} = \frac{1}{3} x^{-2/3} : \frac{d^2y}{dx^2} = -\frac{2}{3} \cdot \frac{1}{3} x^{-5/3} = \boxed{\frac{-2}{9} x^{-5/3}}$$

$$\text{(ii)} \quad y = \frac{x}{x+1} : \frac{dy}{dx} = \frac{(x+1)-x}{(x+1)^2} = \frac{1}{(x+1)^2} : \frac{d^2y}{dx^2} = \boxed{\frac{-2}{(x+1)^3}}$$

$$\text{(iii)} \quad y = x^8 + 5x^6 - 6x^5 + 10x^3 - 2x^2 + 192$$

$$\frac{dy}{dx} = 8x^7 + 30x^5 - 30x^4 + 30x^2 - 4x$$

$$\frac{d^2y}{dx^2} = \boxed{56x^6 + 150x^4 - 120x^3 + 60x - 4}$$

$$\text{(iv)} \quad y = x^{-2} : \frac{dy}{dx} = -2x^{-3} : \frac{d^2y}{dx^2} = 6x^{-4} = \boxed{\frac{6}{x^4}}$$