MATH 1200 2004/5

Homework 7 (Calculus) : Solutions

1. (i) $f(x) = x^3$: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h} = \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0} (3x^2 + 3xh + h^2) = 3x^2 + 0 + 0 = \boxed{3x^2}$$

(ii) $f'(4) = 3\cdot 4^2 = 48$

(iii) Eqn of tangent line is $y - 64 = 48(x-4) = 48x - 192$: $\boxed{y = 48x - 128}$

2. (i) $f'(x) = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{(x+h)^2} - \frac{1}{x^2}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{x^2-(x+h)^2}{(x+h)^2\cdot x^2}\right]$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-2xh-h^2}{x^2(x+h)^2}\right] = \lim_{h\to 0} \frac{-2x-h}{x^2\cdot(x+h)^2} = \frac{-2x}{x^2\cdot x^2} = \boxed{-\frac{2}{x^3}}$$

(ii) $f'(x) = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{x+h+3} - \frac{1}{x+3}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+3)-(x+h+3)}{(x+3)(x+h+3)}\right]$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-h}{(x+3)(x+h+3)}\right] = \lim_{h\to 0} \frac{-1}{(x+3)(x+h+3)} = \boxed{\frac{-1}{(x+3)^2}}$$

(iii) $f'(x) = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right] = \lim_{h\to 0}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]\cdot\frac{1}{h}$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right] = \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \boxed{\frac{-1}{2(\sqrt{x})^3}}$$

square root trick

3. (i) $f(x) = x^7$, $f'(x) = \boxed{7x^6}$ (ii) $f'(x) = \boxed{77x^{76}}$

(iii) $f(x) = x^{-7}$, $f'(x) = \boxed{-7x^{-8}}$ (iv) $f(x) = x^{70}$, $f'(x) = \boxed{70x^{69}}$

(v) $f(x) = x^{1/7}$, $f'(x) = \boxed{\frac{1}{7}x^{-6/7}}$ (vi) $f(x) = \boxed{-\frac{1}{7}x^{-8/7}}$

4. (i) $f(x) = x^{1/11}$; $f'(x) = \boxed{\frac{1}{11}x^{-10/11}}$ (ii) $f(x) = x^{-11}$; $f'(x) = \boxed{-11x^{-12}}$ (iii) $f(x) = x^{2/5}$; $f'(x) = \boxed{\frac{2}{5}x^{-3/5}}$

(iv) $f(x) = x^{-2/5}$; $f'(x) = \boxed{-\frac{2}{5}x^{-7/5}}$ (v) $f(x) = x^{\frac{1}{12}}$; $f'(x) = \boxed{\frac{1}{12}x^{-11/12}}$ (vi) $f(x) = x^{-37/12}$

$f'(x) = \boxed{\frac{-37}{12}x^{-49/12}}$