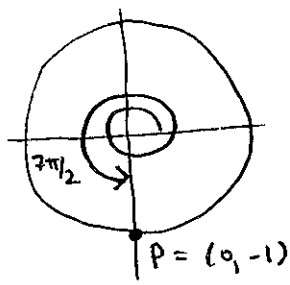
MATH 1200 2004/5

Solutions to Homework 15 (Calculus)

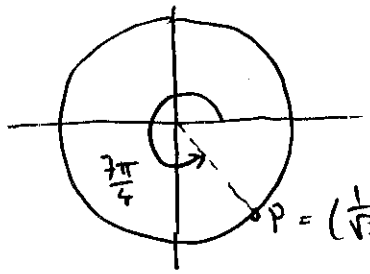
1. (i)



$\cos\frac{7\pi}{2} = 0 \qquad \sin\frac{7\pi}{2} = -1$

$\tan\frac{7\pi}{2} = -\frac{1}{0}$ ← doesn't exist

(ii)

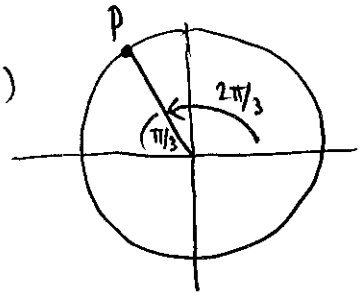


$\cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}}$

$\sin\frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$

$\tan\frac{7\pi}{4} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$

(iii)



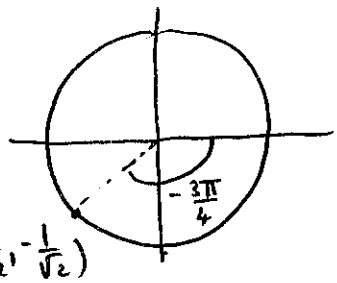
$P = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\cos\frac{2\pi}{3} = -\frac{1}{2}$

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\tan\frac{2\pi}{3} = -\sqrt{3}$

(iv)

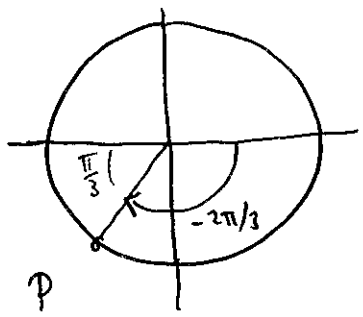


$P = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\cos\left(-\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} = \sin\left(-\frac{3\pi}{4}\right)$

$\tan\left(-\frac{3\pi}{4}\right) = \frac{-1/\sqrt{2}}{-1/\sqrt{2}} = 1$

(v)

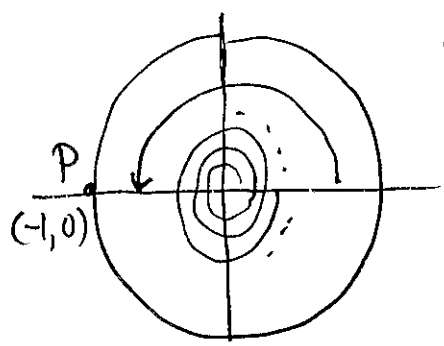


$P = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$: $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$

$\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\tan\left(-\frac{2\pi}{3}\right) = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

(vi)



since 10000003 is odd,

$P = (-1, 0)$.

$\cos(10000003\pi) = -1$

$\sin(10000003\pi) = 0$

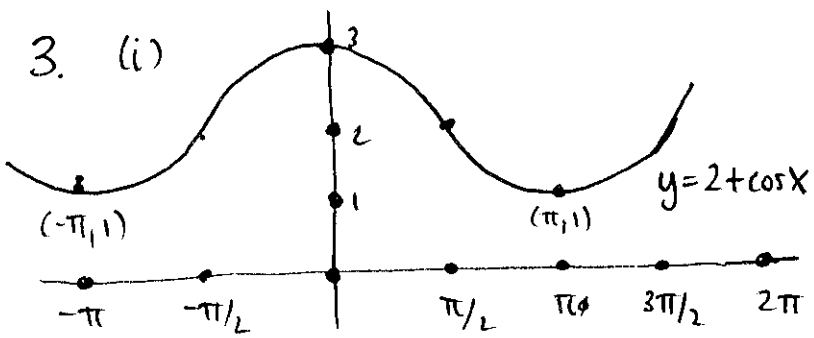
$\tan(10000003\pi) = \frac{0}{-1} = 0.$

2. $1 = \sin^2\theta + \cos^2\theta = \sin^2\theta + \frac{25}{144}$

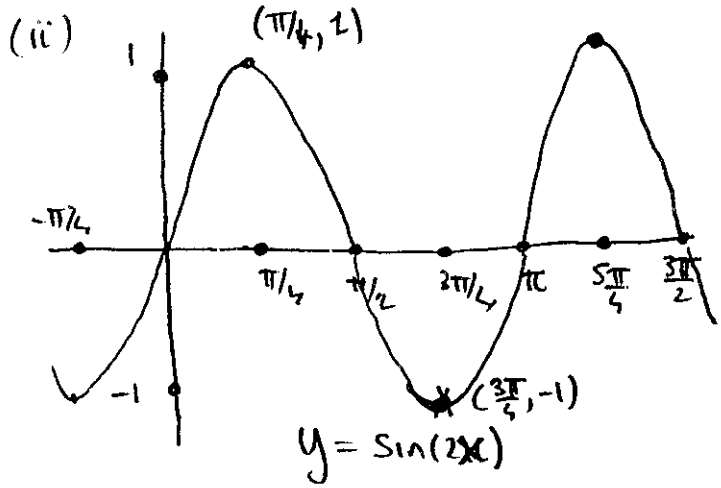
$\therefore \sin^2\theta = 1 - \frac{25}{144} = \frac{119}{144} = \frac{119}{12^2}$.

Since $\sin\theta < 0$, $\sin\theta = -\sqrt{\frac{119}{12^2}} = -\frac{\sqrt{119}}{12}$.

3. (i)



(ii)



4. (i) $\frac{d}{dx}\cos(x^2) = -\sin(x^2)\cdot 2x = -2x\sin(x^2)$

(ii) $\frac{d}{dx}\sin(x^3) = \cos(x^3)\cdot 3x^2 = 3x^2\cos(x^3)$

(iii) $\frac{d}{dx}\cos^2(x) = \frac{d}{dx}\left((\cos x)^2\right) = 2\cos x\cdot -\sin x = -2\cos x\sin x$

(iv) $\frac{d}{dx}(11\sin(5x)) = 11\cos(5x)\cdot 5 = 55\cos(5x)$

(v) $\frac{d}{dx}(\sin x\tan x) = \sin x\cdot(\sec^2 x) + \tan x\cdot\cos x = \sin x\cdot\sec^2 x + \sin x$
$= \sin x(\sec^2 x + 1)$.

(vi) $\frac{d}{dx}\tan(\sqrt{x}) = \sec^2(\sqrt{x})\cdot\frac{1}{2\sqrt{x}} = \frac{\sec^2(\sqrt{x})}{2\sqrt{x}}$

(vii) $\frac{d}{dx}\cot(\sqrt[3]{x}) = -\csc^2(\sqrt[3]{x})\cdot\frac{1}{3}x^{-2/3} = -\frac{\csc^2(\sqrt[3]{x})}{3\sqrt[3]{x^2}}$

5. $P(5) = 150{,}000\cdot(1.04)^5 \approx 182{,}500$

$P(10) = 150{,}000\,(1.04)^{10} \approx 222{,}000$

Doubling time $= d = \frac{\log(2)}{\log(1.04)} \simeq 17.67$ years.

6. $A(t) = 0.35\,b^t$. We have $b^{127} = \frac{1}{2} \Rightarrow b = \left(\frac{1}{2}\right)^{1/127} \approx 0.99456$

Answer: $A(t) = (0.35)\cdot(0.99456)^t$