MATH 1200 2003/4

Solutions to Homework 13 (Calculus)

1. (i) $f(x) = 2x^3 - 3x^2 - 2$; $f'(x) = 6x^2 - 6x = 6 \cdot x(x-1)$

Critical points: 0, 1.

$f(-1) = -2-3-2 = \boxed{-7}$, $f(0) = -2$, $f(1) = -3$, $f(2) = 16-12-2 = \boxed{2}$

smallest　　largest.

So $\underline{-1 \text{ is a global min}}$, $\underline{2 \text{ is a global max}}$

min value is -7　　max. value is 2

(ii)

$f(x) = x^3 - 3x - 1$; $f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$

critical points: $-1, 1$

$f(-2) = -8+6-1 = \boxed{-3}$, $f(-1) = -1+3-1 = \boxed{1}$, $f(1) = \boxed{-3}$, $f(2) = 8-6-1 = \boxed{1}$

-2 and 1 are global minima ; -3 is the minimum value.

-1 and 2 are global maxima ; 1 is the maximum value.

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2. If the price is $10 + x$, then $24 - x$ cars are rented out and the income is

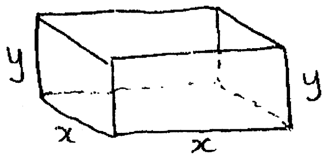
$$I = (10+x)(24-x) = 240 + 14x - x^2$$

This is a quadratic. So it has a global maximum occurring at the unique critical point

$$\frac{dI}{dx} = 14 - 2x \Rightarrow x = 7 \text{ is the critical point}$$

Income is maximized when the price is $10 + 7 = \underline{17 \text{ euro per car}}$

3.



Let x = side of base of box

y = height of box.

Volume is $x^2 y = 4$; $y = 4/x^2$

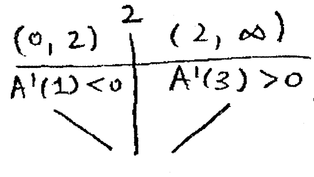
Amount of material is $A = x^2 + 4xy$　　$(x, y > 0)$

$$= x^2 + 4x \cdot \frac{4}{x^2}$$

$$= x^2 + \frac{16}{x} \qquad x > 0.$$

$$\frac{dA}{dx} = 2x - \frac{16}{x^2}; \quad 2x - \frac{16}{x^2} = 0 \Longrightarrow 2x = \frac{16}{x^2} \Longrightarrow x^3 = 8 \Rightarrow x = 2$$

$x = 2$ is the unique critical point.



: $x = 2$ is a $\Rightarrow$ global minimum

Conclusion: A is minimum when $x=2$ any $y=1$.

$x, y > 0$ $\quad x+y=20$; $\quad y=20-x$.

product $= P = xy = x(20-x) = 20x - x^2$

This is a quadratic whose global max occurs at the unique critical point: $\frac{dP}{dx} = 20-2x \Rightarrow x=10$ is the critical point.

Conclusion: The product is maximum when $x=y=10$.

$r = 25.5$, $dr = 0.01$

$V = \frac{4}{3}\pi r^3 \Rightarrow dV = \frac{dV}{dr}dr = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2\,dr = 4\pi r^2 dr$.

So $dV = 4\pi(25.5)^2 \cdot (0.01) \approx 26\pi \approx \underline{81.7\ cm^2}$.

$c = \sqrt{779.5\,T}$ $\qquad T = 280$, $dT = 2$

$dc = \frac{dc}{dT}dT = \frac{1}{2\sqrt{779.5T}} \cdot 779.5\,dT = \frac{1}{2\sqrt{779.5}} \cdot \frac{779.5}{\sqrt{T}}dT$

$= \frac{1}{2\sqrt{779.5}} \cdot \frac{779.5}{\sqrt{280}} \cdot 2 \approx$ ~~0.002 m/s~~ 1.6 m/s