

CHAPTER 4: SOME OTHER FUNCTIONS

1. THE ABSOLUTE VALUE

1.1. The definition and graph of the absolute value.

Definition 1.1. For a real number a , the *absolute value of a* , denoted $|a|$, is the distance from a to 0 on the real line; in other words,

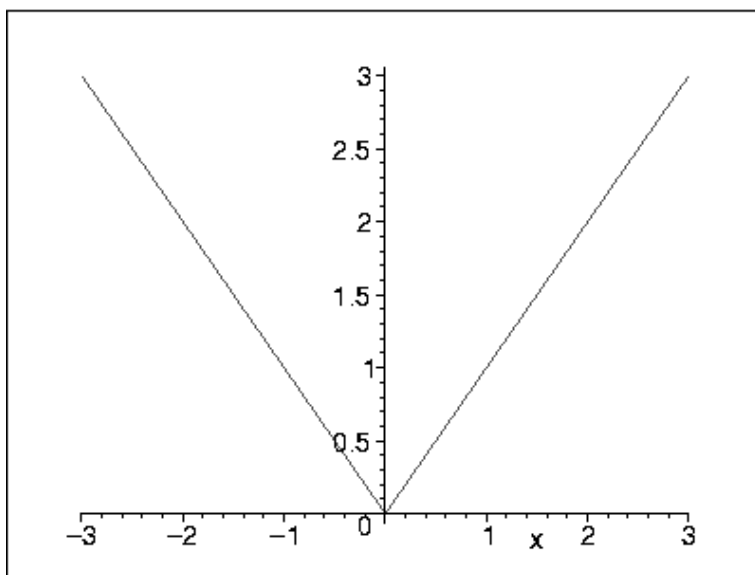
$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

This is a very easy function to evaluate:

Example 1.1. $|5| = 5$, $|3.689| = 3.689$,
 $|-5| = 5$, $|-3.689| = 3.689$, $|\pi| = \pi$, $|0| = 0 \dots$ etc.

When x is a positive real number, $|x| = x$ and thus, for $x > 0$ (that is, to the right of the y -axis) the graph of $f(x) = |x|$ is just the line $y = x$. Similarly, to the left of the y -axis the graph of $f(x) = |x|$ is the line $y = -x$ (with slope -1).

Therefore the graph of the function $f(x) = |x|$ has the following form:



Note that if a and b are any two real numbers then $|a - b|$ = the distance from a to b on the real line.

Example 1.2. $|2 - 5| = |-3| = 3$ = the distance from 2 to 5.
 $|4 - (-3)| = |4 + 3| = |7| = 7$ = the distance from 4 to -3 .

2. SOME FUNCTIONS WITH DISCRETE VALUES

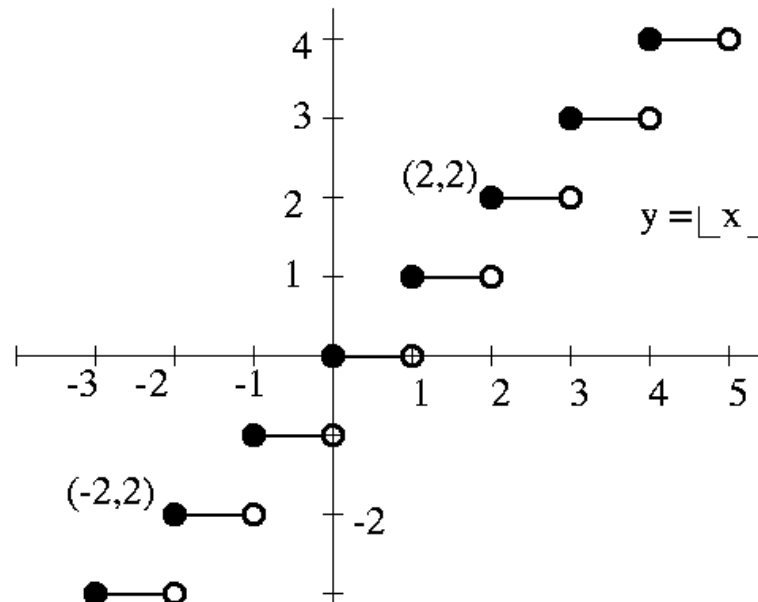
In this section, we introduce some functions whose values are integers. Since the integers are separated from each other on the real line – i.e. they are a ‘discrete’ set – the graphs of such functions tend to have sudden discrete jumps. Functions like this arise naturally in the mathematics of electronic technology.

Definition 2.1. (*The ‘greatest integer function’*): For any real number a , $\lfloor a \rfloor$ denotes the largest integer which is less than or equal to a .

Like the absolute value function, this function is usually easy to evaluate, although a little care needs to be taken with negative numbers:

Example 2.1. $\lfloor 1.5 \rfloor = 1$, $\lfloor 3.782 \rfloor = 3$, $\lfloor -1.5 \rfloor = -2$, $\lfloor 7 \rfloor = 7$, $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4 \dots$ etc.

The graph of $f(x) = \lfloor x \rfloor$ jumps upwards by one unit at each integer input:



Here is a related function:

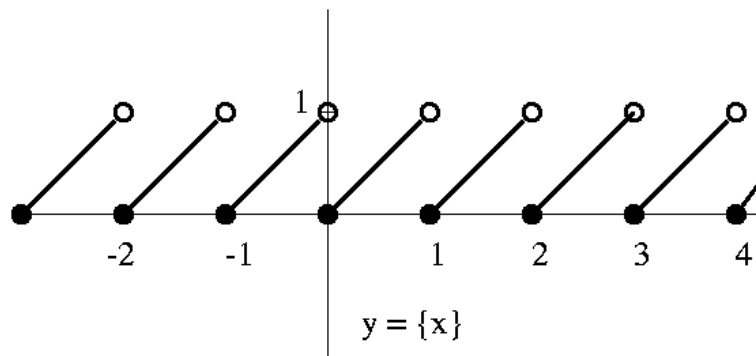
Definition 2.2. (*The ‘fractional part’*):

For any real number a , the *fractional part of a* , denoted $\{a\}$, is $a - \lfloor a \rfloor$.

Example 2.2. $\{3.782\} = 3.782 - 3 = 0.782$

$\{1.5\} = 1.5 - 1 = 0.5$

$\{587.312\} = 0.312 \dots$ etc.



Another integer valued function is the “digitized wave function” : $f(x) = (-1)^{\lfloor x \rfloor}$.

This function outputs the value -1 at x when $\lfloor x \rfloor$ is odd, and the value 1 when $\lfloor x \rfloor$ is even.

For example, $f(7.6) = (-1)^7 = -1$. $f(12.3) = (-1)^{12} = 1$.

