Chapter 2

Matrix Arithmetic

2.1 Introduction to Matrix Arithmetic

Definition 2.1.1 *A $m \times n$ ("m by n") matrix is a matrix having m rows and n columns.*

Example

$$\begin{pmatrix} 2 & 3 & -1 \\ -3 & -4 & 0 \end{pmatrix}$$ is a 2×3 matrix.

$$\begin{pmatrix} 2 & 3 \\ 2 & 7 \\ 4 & 0 \end{pmatrix}$$ is a 3×2 matrix.

Definition 2.1.2 *Two matrices are said to have the same* size *if they have the same number of rows and the same number of columns. (So for example a 3×2 matrix and a 2×3 matrix are considered to be of different size).*

Notation: If A is an $m \times n$ matrix, the entry appearing in the ith row and jth column of A (called the (i,j) position) is denoted $(A)_{ij}$.

Example: Let $A = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 0 & 5 \end{pmatrix}$.

Then $(A)_{11} = 2,\ (A)_{21} = 4,\ (A)_{13} = -1$, etc.

Remark: A matrix having just one row is sometimes called a *row vector*, and a matrix having just one column is called a *column vector*.

Definition 2.1.3 Matrix Addition. *LetA and B be matrices of the same size ($m \times n$). We define their* sum $A + B$ *to be the $m \times n$ matrix whose entries are given by*

$$(A + B)_{ij} = (A)_{ij} + (B)_{ij}$$

for $i = 1, \ldots, m$ and $j = 1, \ldots, n$

Thus $A + B$ is obtained from A and B by adding entries in corresponding positions.

Example: Let $A = \begin{pmatrix} 2 & 0 & -1 & -1 \\ 1 & 2 & 4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 1 & 0 & -2 \\ 3 & -3 & 1 & 1 \end{pmatrix}$. Then

$$A + B = \begin{pmatrix} 2 + (-1) & 0 + 1 & -1 + 0 & -1 + (-2) \\ 1 + 3 & 2 + (-3) & 4 + 1 & 2 + 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & -3 \\ 4 & -1 & 5 & 3 \end{pmatrix}$$

Subtraction of matrices is now defined in the obvious way - e.g., with A and B as above, we have

$$A - B = \begin{pmatrix} 2 - (-1) & 0 - 1 & -1 - 0 & -1 - (-2) \\ 1 - 3 & 2 - (-3) & 4 - 1 & 2 - 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 & -1 & 1 \\ -2 & 5 & 3 & 1 \end{pmatrix}$$

Definition 2.1.4 Multiplication of a Matrix by a Real Number

Let A be a $m \times n$ matrix and let c be a real number. Then cA is the $m \times n$ matrix with entries defined by

$$(cA)_{ij} = c(A)_{ij}$$

i.e. cA is obtained from A by multiplying every entry by c.

Example: If $A = \begin{pmatrix} 2 & 1 \\ 3 & -4 \end{pmatrix}$, then

$$2A = \begin{pmatrix} 4 & 2 \\ 6 & -8 \end{pmatrix}, \ -3A = \begin{pmatrix} -6 & -3 \\ -9 & 12 \end{pmatrix}, \ 0A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Definition 2.1.5 *The $m \times n$ matrix whose entries are all zero is called the* zero *($m \times n$) matrix.*