MATH1200 Algebra
Homework 16

1. Find the area of each of the following objects in $\mathbb{R}^3$.

   (a) The triangle with vertices $A(3,-1,2)$, $B(4,2,1)$ and $C(5,1,-3)$.

   (b) The parallelogram having the vectors $\vec{u} = [3,-1,4]$ and $\vec{v} = [6,2,-2]$ as adjacent sides.

   (c) The triangle with vertices $A(1,0,2)$, $B(3,-2,3)$ and $C(5,0,-1)$.

   (d) The parallelogram having the vectors $\vec{u} = [2,2,-1]$ and $\vec{v} = [-1,2,4]$ as adjacent sides.

2. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ denote the vectors $[2,0,4]$, $[1,-3,-2]$ and $[1,2,2]$. Find the volume of the parallelepiped having $\vec{u}$, $\vec{v}$ and $\vec{w}$ as adjacent edges.

3. Let $\vec{u} = [2,-3,2]$, $\vec{v} = [1,0,-1]$ and $\vec{w} = [1,2,-2]$. Verify by direct computation that the scalar triple product $\vec{u}.(\vec{v} \times \vec{w})$ is equal to the determinant of the 3×3 matrix having the components of $\vec{u}, \vec{v}$ and $\vec{w}$ respectively as the entries of its three rows.

4. Find a vector equation for the line L in $\mathbb{R}^3$ containing the points $A(2,-5,7)$ and $B(1,-2,3)$. Determine which of the following points belongs to L.

   (i) $(-1,4,-5)$

   (ii) $(5,-14,18)$

   (iii) $(6,-7,23)$

5. For each of the following pairs of lines in $\mathbb{R}^3$, either find their intesection point or determine that no intersection point exists.

   (a) $L_1 : \vec{r}(t) = [-3,1,5] + t[1,2,-4],\ t \in \mathbb{R},$
   $L_2 : \vec{m}(s) = [8,-1,0] + s[3,-2,1],\ s \in \mathbb{R}.$

   (b) $L_1 : \vec{r}(t) = [2,1,-2] + t[4,-2,-5],\ t \in \mathbb{R},$
   $L_2 : \vec{m}(s) = [2,7,-5] + s[2,-3,-1],\ s \in \mathbb{R}.$

   (c) $L_1 : \vec{r}(t) = [3,-2,4] + t[1,5,-3],\ t \in \mathbb{R},$
   $L_2 : \vec{m}(s) = [-4,-1,1] + s[-1,-2,3],\ s \in \mathbb{R}.$

Answers overleaf

Answers:

1. (a) $\frac{1}{2}\sqrt{194}$
   (b) $10\sqrt{10}$
   (c) $5\sqrt{2}$
   (d) $\sqrt{185}$
2. 16
3. 5
4. (i) Yes
   (ii) No
   (iii) No
5. (a) $(-1, 5, -3)$
   (b) No intersection point
   (c) No intersection point