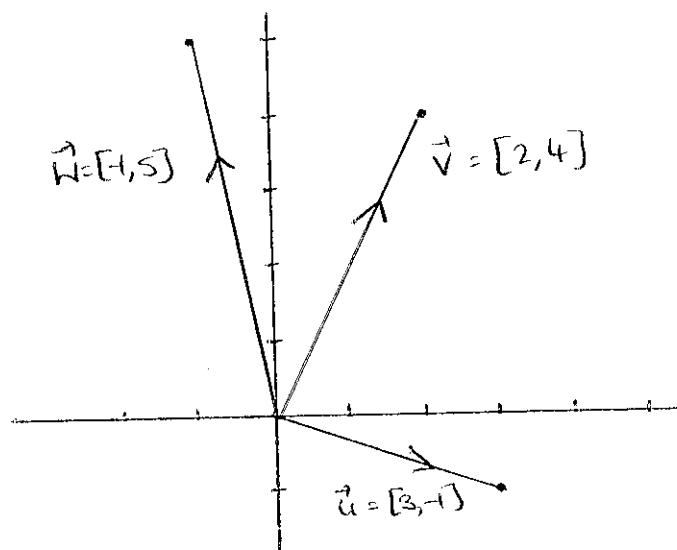


Solutions to Homework 10

1.



2. (a) $\vec{u} + \vec{v} = [3, -1] + [2, 4] = [5, 3]$

(b) $2\vec{u} + \vec{v} = 2[3, -1] + [2, 4]$
 $= [6, -2] + [2, 4]$
 $= [8, 2]$

(c) $\vec{u} + \vec{v} + \vec{w} = [3, -1] + [2, 4] + [-1, 5]$
 $= [4, 8]$

(d) $\vec{v} - 2\vec{w} = [2, 4] - 2[-1, 5] = [2, 4] + [2, -10]$
 $= [4, -6]$

3. (a) $\|[3, -4]\| = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

(b) $\|[-\frac{3}{4}, 1]\| = \sqrt{(-\frac{3}{4})^2 + 1^2} = \sqrt{\frac{9}{16} + 1} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

(c) $\|[6, -5]\| = \sqrt{6^2 + (-5)^2} = \sqrt{36 + 25} = \sqrt{61}$

$$4. \quad \vec{v} = [2, -5]$$

$$(a) \quad \|\vec{v}\| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$$

Unit vector in direction of \vec{v} :

$$\hat{v} = \frac{1}{\sqrt{29}} \vec{v} = \frac{1}{\sqrt{29}} [2, -5] = \left[\frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}} \right]$$

(b) Vector of length $\frac{1}{2}$ in direction of \vec{v} :

$$\frac{1}{2} \hat{v} = \frac{1}{2} \frac{1}{\sqrt{29}} [2, -5]$$

$$= \left[\frac{1}{\sqrt{29}}, -\frac{5}{2\sqrt{29}} \right]$$

(c) A vector of length 4 with direction opposite to that of \vec{v} is

$$-4 \hat{v} = -4 \frac{1}{\sqrt{29}} [2, -5]$$

$$= \left[\frac{-8}{\sqrt{29}}, \frac{20}{\sqrt{29}} \right]$$

$$5. \quad (a) \quad \vec{u} \cdot \vec{v} = [1, 2] \cdot [-4, 2] = -4 + 4 = 0 \Rightarrow \vec{u} \perp \vec{v}$$

$$(b) \quad \vec{u} \cdot \vec{v} = [1, 2] \cdot [-2, -1] = -2 - 2 = -4 \Rightarrow \vec{u} \not\perp \vec{v}$$

(\vec{u} and \vec{v} are not orthogonal)

$$(c) \quad \vec{u} \cdot \vec{v} = [3, 5] \cdot [2, -6] = 6 - 30 = -24$$

$$\Rightarrow \vec{u} \not\perp \vec{v}$$

(not orthogonal).