

MST 10010: Calculus I

Midterm review

1) Let $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (x - 3)^2 & \text{if } x > 3. \end{cases}$

a) Evaluate each limit, if it exists.

(i) $\lim_{x \rightarrow 0^+} f(x)$

(ii) $\lim_{x \rightarrow 0^-} f(x)$

(iii) $\lim_{x \rightarrow 0} f(x)$

(iv) $\lim_{x \rightarrow 3} f(x)$

b) Where is f discontinuous and why?

2) Evaluate the following limits, if they exist.

a) $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$

b) $\lim_{x \rightarrow \infty} \frac{x^4 - 3x}{x^4 + 100x^3 + x^2 + x + 7}$

3) State the definition for a function f to be continuous at a .

4) If $3x \leq f(x) \leq x^3 + 2$ for all x in the interval $[0, 2]$, then find $\lim_{x \rightarrow 1} f(x)$.

5) Show that there is a root of the equation $2x^3 + x^2 + 2 = 0$ in the interval $(-2, -1)$.

6) Find all asymptotes of the curve $f(x) = \frac{4-x}{x+3}$.

7) Find the equation of the tangent line to the curve $f(x) = x^2 + 2x + 1$ at the point where $x = 1$.

8) Use the definition of the derivative to find $f'(x)$ if $f(x) = \sqrt{3-5x}$.

9) True or False:

a) If $\lim_{x \rightarrow 0} f(x) = \infty$ and $\lim_{x \rightarrow 0} g(x) = \infty$, then $\lim_{x \rightarrow 0} [f(x) - g(x)] = 0$.

b) If f is continuous at a , then f is differentiable at a .

c) If $f(1) > 0$ and $f(3) < 0$, then there exists a number c between 1 and 3 such that $f(c) = 0$.

d) If $p(x)$ is a polynomial, then $\lim_{x \rightarrow a} p(x) = p(a)$.

10) Given $H(x) = (x^3 - x + 1)(x^{-2} + 2x^{-3}) + \frac{x}{\sin x + \cos x}$, find $H'(x)$.