Last time: Infinite limits

Squeeze Theorem.

Today: Go back ...

EX] $\lim_{x \to 2} x^2+1 = (2)^2+1 = 5.$

NOTE: Polynomial $P(x)$ have the nice property that

$$\lim_{x \to a} P(x) = \underbrace{P(a)}_{\text{sub } x=a \text{ into } P(x).}$$

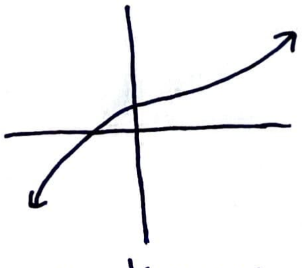
Question: What other functions have this property?

Ans: "Continuous functions"

(no breaks or jumps in the graph of $f(x)$

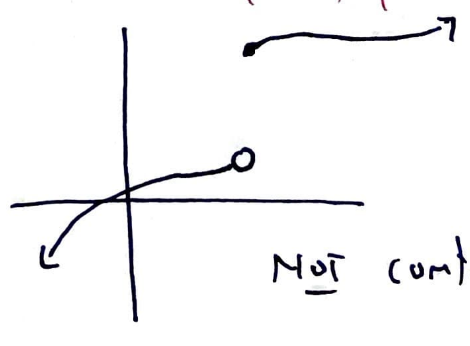
Drawing a graph → "don't lift the pen/pencil")

EX]



continuous

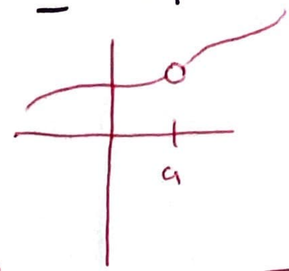
EX]



NOT continuous.

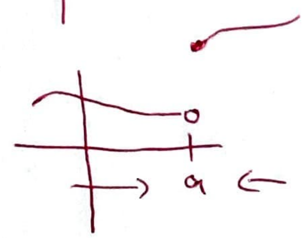
Def. A function f is continuous at a if

i) $f(a)$ exists (takes care of



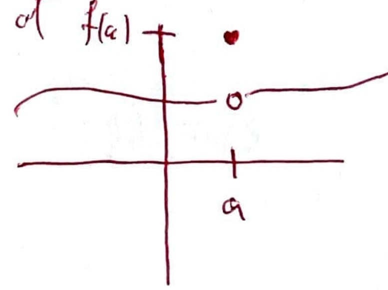
)

ii) $\lim_{x \to a} f(x)$ exists (takes care of



)

iii) $\lim_{x \to a} f(x) = f(a)$ (takes care of $f(a)$



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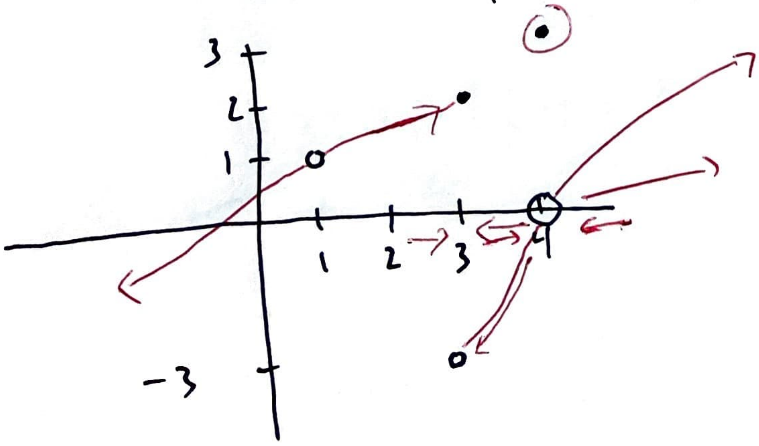
Notes: ① This def. is equivalent to saying

$$\lim_{x \to a} f(x) = f(a).$$

② If f is not continuous at a, we say f is discontinuous at a.

EX] Where is this function discontinous and why?



$f(x)$

$\lim_{x \to 4} f(x) = 0$

$f(4) = 3$

Discont. at $\underline{x=1}$: Why? $f(1)$ DNE $\Rightarrow$ (i) fails.

Discont. at $\underline{x=3}$: Why? $\lim\limits_{x\to 3} f(x)$ DNE $\Rightarrow$ (ii) fails

Why not?

$$\lim_{x\to 3^+} f(x) = -3$$

$$\lim_{x\to 3^-} f(x) = 2$$

Discont. at $x=\underline{4}$: Why?

$$\lim_{x\to 4} f(x) = 0, \quad \text{but } f(4) = 3$$

So, $\lim\limits_{x\to 4} f(x) \neq f(4)$

$\Rightarrow$ (iii) fails.

EX] Given

$$f(x) = \begin{cases} \dfrac{x^2-2x-8}{x-4} & \text{if } x \neq 4 \\ 3 & \text{if } x = 4. \end{cases}$$

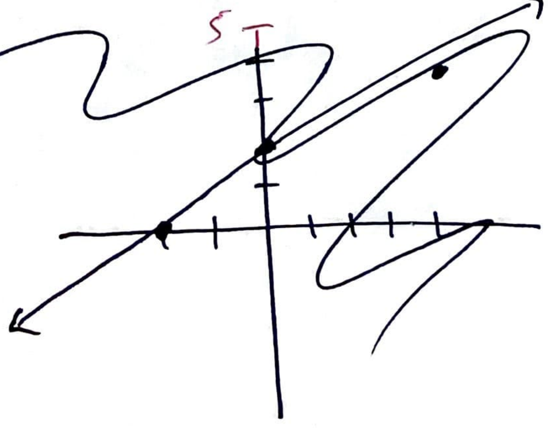
Explain why it is discont. at $a=4$.

NOTE: if $x\neq 4$, $f(x) = \dfrac{x^2-2x-8}{x-4} = \dfrac{\cancel{(x-4)}(x+2)}{\cancel{x-4}}$

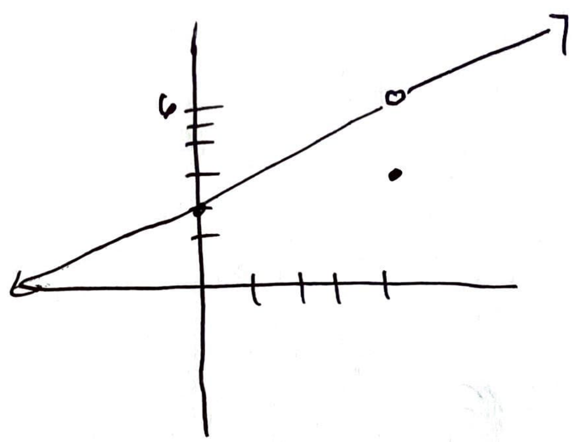
$= x+2$ (line)

if $x=4$, $f(4)=3$.

Graph:



Graph:



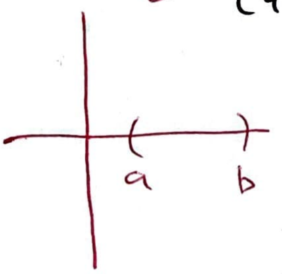
Check: i) $f(4) = 3$ ✓ Exists

ii) $\lim_{x \to 4} f(x) = 6$ ✓ Exists

iii) $\lim_{x \to 4} f(x) = 6 \neq 3 = f(4)$.

Generalize!

Def. A function f is continuous on an interval (a,b) if it is continuous at every point in (a,b).



NOTE: Left endpoint needs continuous from right i.e. $[a, b)$

Right endpoint need continuous from left i.e. $(a, b]$.

Clarify this!

Def. A function f is continuous from the left (respectively, right) at a if

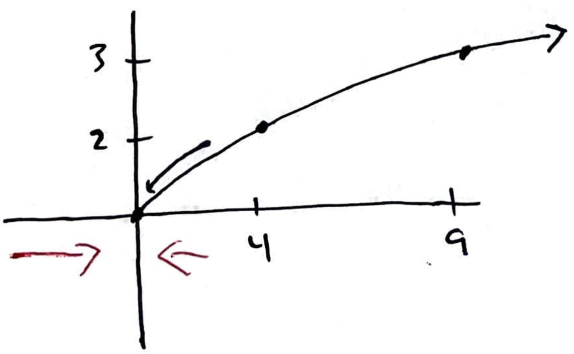
i) $f(a)$ exists

ii) $\lim_{x \to a^-} f(x) = f(a)$ (respectively, $\lim_{x \to a^+} f(x) = f(a)$)

EX] Consider $f(x) = \sqrt{x}$.

Is f cont. from the right or left at $a = 0$?

On what interval is f cont.?



i) $f(0) = 0$ ✓

ii) $\lim_{x \to 0^-} f(x)$ DNE

$\lim_{x \to 0^+} f(x) = 0 = f(0)$

Continuous from the right at 0.

In fact, f is cont. on $[0, \infty)$ ✓

From the limit rules, we get ...