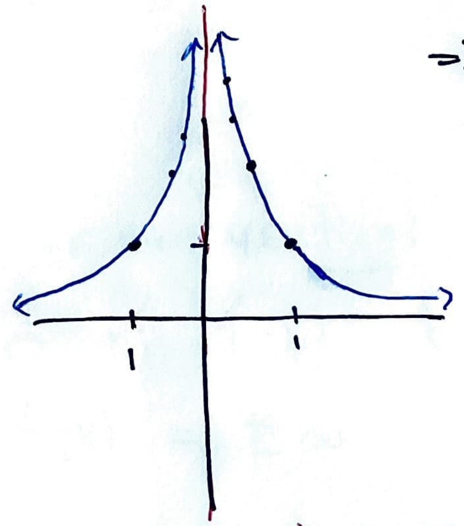
Last time: composition of functions
Limits.

TODAY: infinite limits
Limit Laws

Infinite Limits:

EX] $y = \frac{1}{x^4}$

x	y
1	1
.5	16
.1	10,000
.01	100,000,000
-1	1
-.5	16
-.1	10,000
⋮	⋮

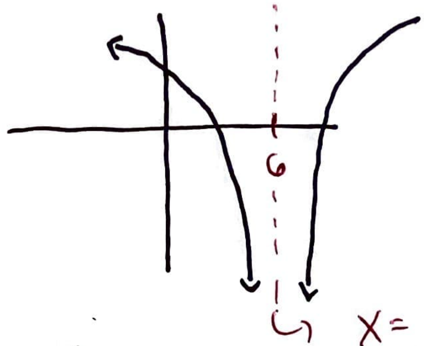


⇒ $f(x)$ gets larger w/o bound

" the limit of $\frac{1}{x^4}$, as x approaches 0, is ∞
("infinity")

$$\lim_{x \to 0} \frac{1}{x^4} = +\infty$$

Note: ① Suppose we have



$f(x)$

then $\lim\limits_{x \to 6} f(x) = -\infty$.

② In the example with $y = \frac{1}{x^4}$, the graph approached the line $x = 0$. Such a line is an example of ..

Def. A line $x = a$ is a vertical asymptote (V.A.) of the graph of $f(x)$ if either

$$\lim_{x \to a^+} f(x) = \pm\infty$$

or

$$\lim_{x \to a^-} f(x) = \pm\infty .$$

Note: V.A.'s will play an important role in "Curve-sketching".

EX] Compute $\displaystyle\lim_{x \to 0^+} \frac{1}{x^4}$.

Could pick values of x to the right of 0 and substitute (as above)

OR

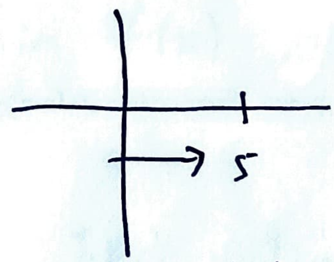
$\dfrac{1}{x^4}$ small positive

⇓

whole thing is a large positive number.

$\Rightarrow$ So: $\displaystyle\lim_{x \to 0^+} \frac{1}{x^4} = +\infty$.

EX] Compute $\displaystyle\lim_{x \to 5^-} \frac{6}{x-5}$



Pick values to the left of 5 and substitute

OR

large negative number $\Leftarrow$ $\dfrac{6}{x-5}$ small negative

$\Rightarrow$ So: $\displaystyle\lim_{x \to 5^-} \frac{6}{x-5} = -\infty$.

Limit Laws:

Idea: Let us use some rules to help calculate limits.

Rule 1: $\lim\limits_{x \to a} [f(x) + g(x)] = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$.

Rule 2: $\lim\limits_{x \to a} [f(x) - g(x)] = \lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x)$.

Rule 3: $\lim\limits_{x \to a} [c f(x)] = c \lim\limits_{x \to a} f(x)$.

Rule 4: $\lim\limits_{x \to a} [f(x) g(x)] = \lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)$.

Rule 5: $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$

if $\lim\limits_{x \to a} g(x) \neq 0$

Rule 6: $\lim\limits_{x \to a} [f(x)]^n = \left[\lim\limits_{x \to a} f(x)\right]^n$

where n is a positive integer.

Rule 7: $\lim_{x \to a} c = c$ where c is a real number.

(can you see this from the graph $y = c$?)

Rule 8: $\lim_{x \to a} x^n = a^n$, n positive integer.

EX] Compute $\lim_{x \to 3} (x^3+2)(x^2-5x)$ (Rule 4)

So: $\lim_{x \to 3} (x^3+2)(x^2-5x) = \lim_{x \to 3} (x^3+2) \cdot \lim_{x \to 3} (x^2-5x)$

$$= \left(\lim_{x \to 3} x^3 + \lim_{x \to 3} 2\right)\left(\lim_{x \to 3} x^2 - \lim_{x \to 3} 5x\right)$$

Rules 1, 2

$$= (27 + 2)(9 - 15) \quad \text{(Rules 3, 7, 8)}$$

$$= -174.$$

EX] Compute $\lim_{x \to -2} \dfrac{x^3 + 2x^2 - 1}{5 - 3x}$

$$= \frac{\lim_{x \to -2} x^3 + 2x^2 - 1}{\lim_{x \to -2} 5 - 3x}$$

$$= \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = -\frac{1}{11}.$$

NOTE: We can use Rule 5 since $\sqrt[3]{5}$

$$\lim_{x \to -2} 5 - 3x = 11 \neq 0.$$

$\sqrt[2]{0}, \sqrt[4]{0}, \sqrt[6]{0}$

Rule 9: $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$, n positive ~~positive~~ integer

($a > 0$ if n is even)

Rule 10: $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$

n positive integer

($\lim_{x \to a} f(x) > 0$ if n is even).

- Question: Can we always just substitute $x = a$ to calculate $\lim_{x \to a} f(x)$?

Not always! EX] $\lim_{x \to 0} \frac{\sin x}{x}$ (Later)

$= ?$

Important Fact: If $f(x)$ is a polynomial or a rational function (ratio of polys) and a is in the domain of $f(x)$, then

$$\lim_{x \to a} f(x) = f(a).$$