Last time: Examples of functions
Piecewise functions, trig. functions

Today: Creating new functions.
Limits

Creating new functions

Question: How can we create a new function from two functions f, g?

(add, subtract, Multiply, Divide)

Def. Given two functions f and g, the composition of f and g is given by

$$(f \circ g)(x) = f(\underbrace{g(x)})$$

"f is composed with g"

compute $g(x)$ first

then compute f

EX] If $f(x) = x^2$, $g(x) = x-3$, compute $(f \circ g)(x)$, $(f \circ g)(4)$, $(g \circ f)(x)$, $(g \circ f)(4)$.

So: $(f\circ g)(x) = f(g(x)) = f(x-3) = (x-3)^2$ ✓

Thus: $(f\circ g)(4) = (4-3)^2 = (1)^2 = 1.$

Also, $(g\circ f)(x) = g(f(x)) = g(x^2) = x^2 - 3.$ ✓

$\Rightarrow (g\circ f)(4) = 4^2 - 3 = 13.$

In general: $f\circ g \neq g\circ f$.

Ex] If $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$, find each function and its domain.

(i) $f\circ g$

(ii) $g\circ g$.

Solution: (i) $(f\circ g)(x) = f(g(x)) = f(\sqrt{2-x})$

$$= \sqrt{\sqrt{2-x}}$$

$$= \sqrt[4]{2-x}.$$

Domain = all real numbers $x \leq 2$

$= (-\infty, 2]$

can't take fourth roots of negative #'s.

$\Rightarrow 2 - x \geq 0.$

$\Rightarrow x \leq 2.$

(ii) $(g\circ g)(x) = g(g(x))$

$= g(\sqrt{2-x}) = \sqrt{2-\sqrt{2-x}}$

defined if

$2 - x \geq 0$

$\Rightarrow x \leq 2$

In total,

Domain $= -2 \leq x \leq 2$

$= [-2, 2]$.

defined if

$2 - \sqrt{2-x} \geq 0$

$\Rightarrow 2 \geq \sqrt{2-x}$

$\Rightarrow$ square both sides to get

$4 \geq 2 - x$

$\Rightarrow x \geq -2$.

Ex] Given $f(x) = \dfrac{x}{x+1}$,

$g(x) = x^{10}$, $h(x) = x+3$,

compute $f \circ g \circ h$.

Solution: $f \circ g \circ h(x) = f(g(h(x)))$

$= f(g(x+3))$

$= f((x+3)^{10})$

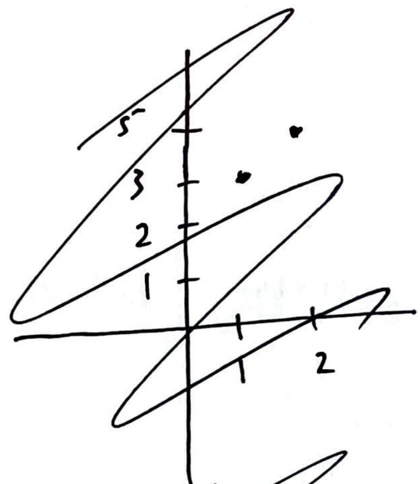
$= \dfrac{(x+3)^{10}}{(x+3)^{10} + 1}$.

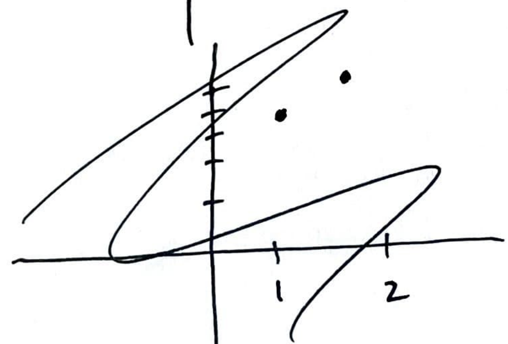
The Limit of a Function:

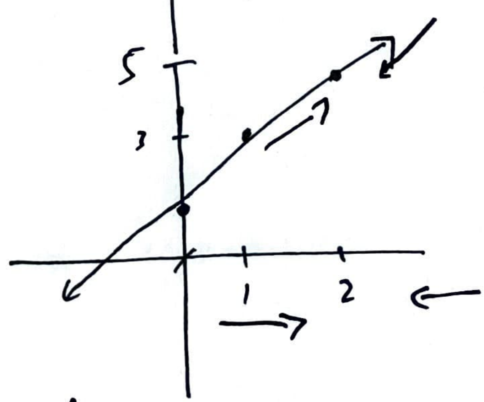
Question: What do we mean by the limit of a function?

EX] $f(x) = 2x + 1$

X	Y
1	3
1.5	4
1.9	4.8
2.5	6
2.1	5.2
2.01	5.02
2.001	5.002







We say: "the limit of $f(x)$, as x approaches 2, equals 5".

Notation: $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} 2x+1 = 5.$

In general,

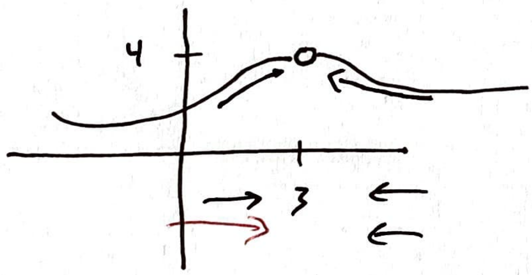
Def. We write $\lim_{x \to a} f(x) = L$ and

say "the limit of $f(x)$ as x approaches a, equals L".

Note: ~~We don't actually~~

We care about what happens "near a".

EX]

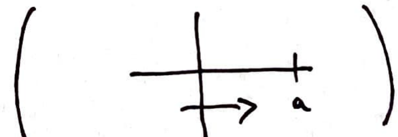


o = means the point $(3,4)$ is not on the graph.

$$\lim_{x \to 3} f(x) = 4$$

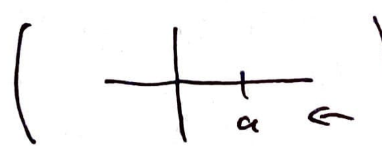
Def. We write $\lim_{x \to a^-} f(x) = L$ and say

"the limit of $f(x)$, as x approaches a from the left, is L"



Def. We write $\lim_{x \to a^+} f(x) = L$ and say

"the limit of $f(x)$, as x approaches a from the right, is L"



NOTE: ① We have $\lim_{x \to a} f(x) = L$

if and only if

$(\Leftrightarrow)$

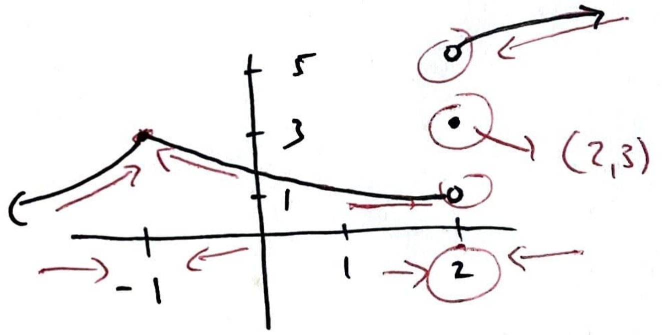
$\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$.

② If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then

$\lim_{x \to a} f(x)$ does not exist (DNE).

(must check Left and Right limits).

EX]



a) find $\lim_{x \to 2} f(x)$

$\left(\lim_{x \to 2^+} f(x) = 5 \neq \lim_{x \to 2^-} f(x) = 1 \Rightarrow \text{DNE} \right)$

b) find $\lim_{x \to -1} f(x)$

$\left(\lim_{x \to -1^-} f(x) = 3 \quad \lim_{x \to -1^+} f(x) = 3 \Rightarrow \lim_{x \to -1} f(x) = 3 \right)$

c) find $f(2)$

$f(2) = 3$

NEXT TIME:

Infinite Limits.