

# MAT 10010: Calculus I

Overview: • Functions, Limits and Derivatives

• Differentiation Rules  
(standard rules, implicit diff.)

• Applications of Differentiation  
(max/min, curve sketching, L'Hôpital's Rule)

• Integrals (via Riemann sums)

## Section 1: Functions.

Question: What is a function?

Def. A function  $f$  is a rule that assigns to each element in a set  $A$  a unique element  $f(x)$  in a set  $B$ .

Note: (1) The set  $A$  is called the domain of  $f$

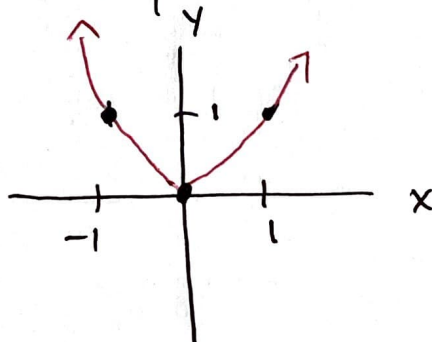
(all possible "inputs")

(2) If  $x$  is an element of  $A$ , then  $f(x)$  is the value of  $f$  at  $x$  (ONE output) ("f of x")

(3) The set of all such values is the range of  $f$  (all possible outputs)

EX] Consider  $f(x) = x^2$  (parabola)

X	Y
0	$(0)^2 = 0$
1	$(1)^2 = 1$
$(-1)$	$(-1)^2 = 1$



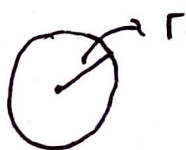
Domain =  $\mathbb{R}$  (set of all real numbers)

Note that  $x^2 \geq 0$  for any real number  $x$

$\Rightarrow$  Range = all real numbers  $y \geq 0$

=  $[0, \infty)$  ("interval notation")

EX] "The area of a "circle" is  $\pi r^2$ "



$$\Rightarrow A(r) = \pi r^2$$

Domain = all real numbers  $\geq 0$

Range = all real numbers  $\geq 0$ .

## Piecewise Functions:

Idea: We could have different formulas for different values of a domain.

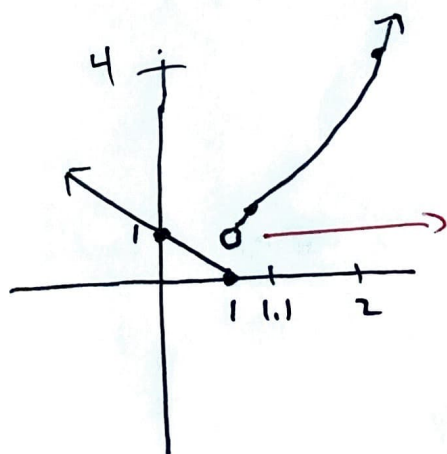
EX] Consider

$$f(x) = \begin{cases} 1-x & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

This means: if  $x \leq 1 \Rightarrow f(x) = 1-x$  (line)  
if  $x > 1 \Rightarrow f(x) = x^2$  (parabola)

So:

X	Y
0	$1-0=1$
1	$1-1=0$
1.1	$(1.1)^2 = 1.21$
2	$(2)^2 = 4$



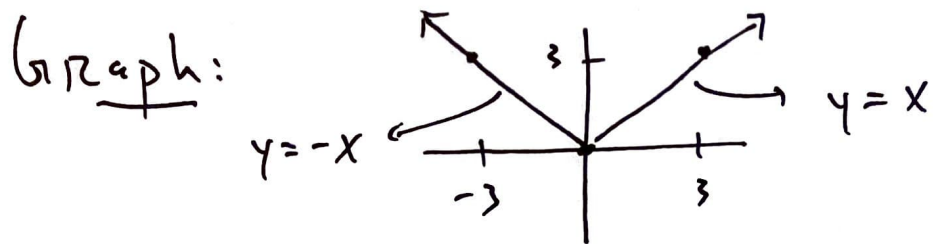
means that the point (1,1) is not on the graph.

EX] For any real number  $x$ ,  
consider

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

("absolute value function")

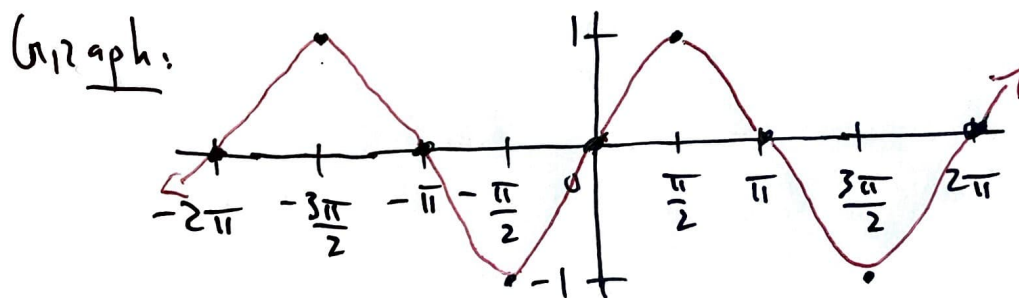
So:  $|3| = 3$ ,  $|-3| = -(-3) = 3$   
 $|0| = 0$ .



## Trigonometric Functions:

Recall these trig. functions..

EX]  $f(x) = \sin x$  ("sine function")



Domain =  $\mathbb{R}$

Range =  $[-1, 1]$ . Thus, we have

$$-1 \leq \sin x \leq 1$$

for all real numbers  $x$ .

EX]  $f(x) = \cos x$  ("cosine function")

Graph (Do this!)

(check that Domain =  $\mathbb{R}$

$$\text{Range} = [-1, 1]$$

$$\Rightarrow -1 \leq \cos x \leq 1$$

for all real  $x$ .