Black Holes in VR

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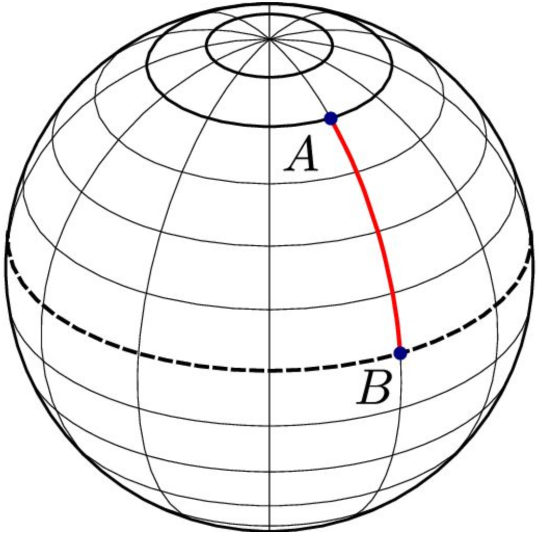
Introduction to General Relativity

- Fundamentally different to Newton's law of Gravitation
- Gravity is a manifestation of **curved spacetime**
- Spacetime described by a **Riemann Manifold** with a **metric g_{uv}**
- Metric is needed to calculate distance, angles and volume in curved space
- Flat space → **Minkowski Metric $\eta_{\mu\nu}$** → Special Relativity

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

How do objects move in Curved Spacetime?

- Flat space → Straight Lines
- Curved space shortest distance represented by **Geodesics**



- **Parallel transport** of tangent vector gives us the geodesic equation

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = 0$$

$\Gamma^\lambda_{\mu\nu}$ - Christoffel coefficient

What defines the Metric?

Newton's gravitational field

$$\nabla^2\phi = 4\pi G\rho,$$

Einstein's Field Equations

$$G^{\alpha\beta} = 8\pi T^{\alpha\beta}.$$

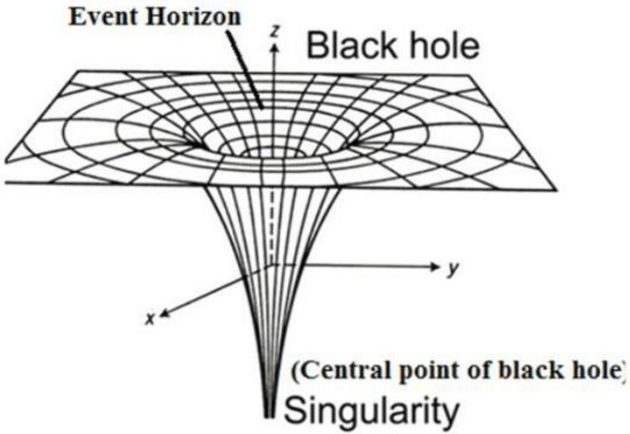
- **Stress Energy Tensor** ***T*** causes the curvature of spacetime

Define **T** → Use Einstein Eq. to get $\mathbf{g}_{\mu\nu}$ → Use metric to find Γ → Use Γ to solve geodesics

- There exist exact solutions to the Einstein Field Equation (eg: Schwarzschild solution)

What is a Black Hole?

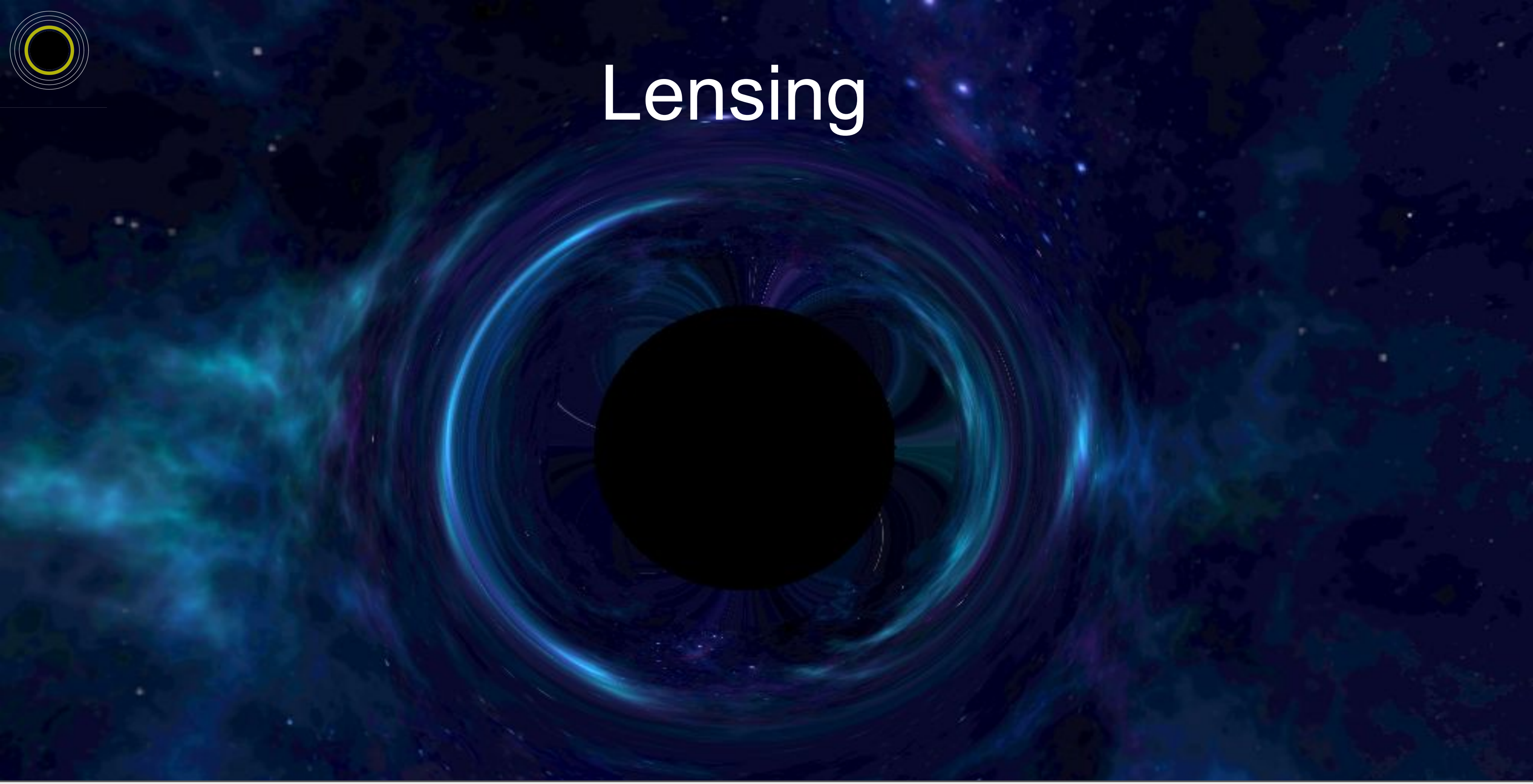
- Region of spacetime so deformed even light cannot escape
- Boundary of no escape - **Event Horizon**
- **Singularity** is where curvature become infinite



$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}dtd\phi + \frac{\Sigma}{\rho^2}\sin^2\theta d\phi^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2.$$

- **Kerr** black hole → **rotating**, uncharged and axially symmetric
- Introduces novel effects such as frame dragging

Lensing



Gravitational Lensing

- Light travels along **Null-Geodesics**

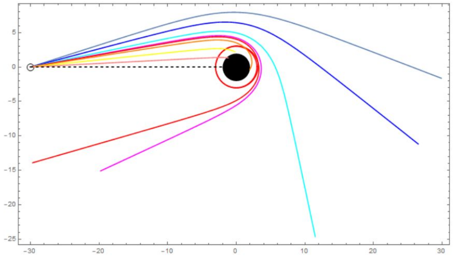
$$\dot{x}^\mu \dot{x}_\mu = \begin{cases} -1 & \text{for timelike geodesics} \\ 0 & \text{for null geodesics} \\ 1 & \text{for spacelike geodesics} \end{cases},$$

- Find Lagrangian, $\mathscr{L} = \mathcal{H}$ and apply the Hamiltonian equations of motion

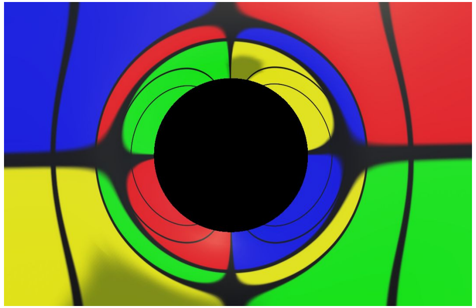
$$\mathscr{L} = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu .$$

$$\dot{x}^\mu = \frac{\partial \mathscr{L}}{\partial p_\mu}, \dot{p}_\mu = -\frac{\partial \mathscr{L}}{\partial x^\mu}.$$

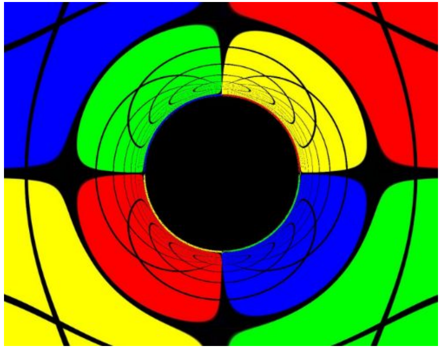
- This combined with $\mathscr{L} = 0$ gives us first order differential equations which we can solve



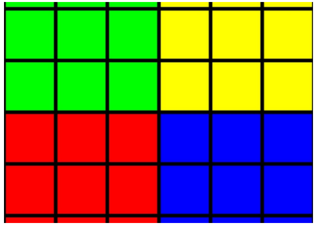
Shader Approximation



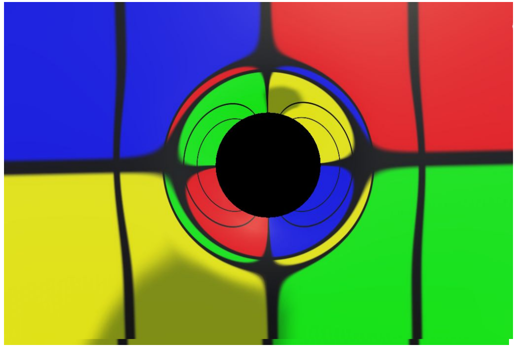
Shader in VR Project



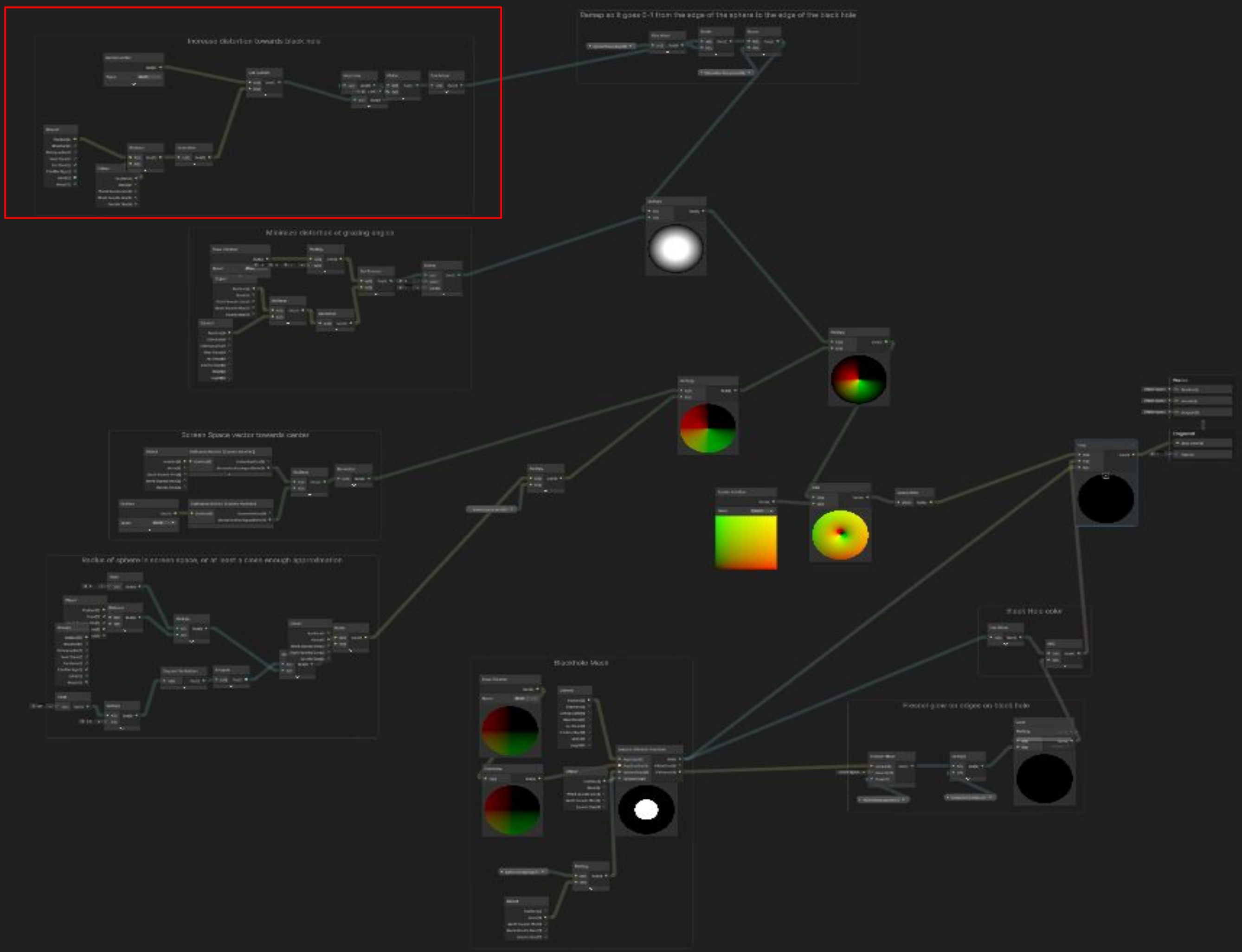
Correct lensing



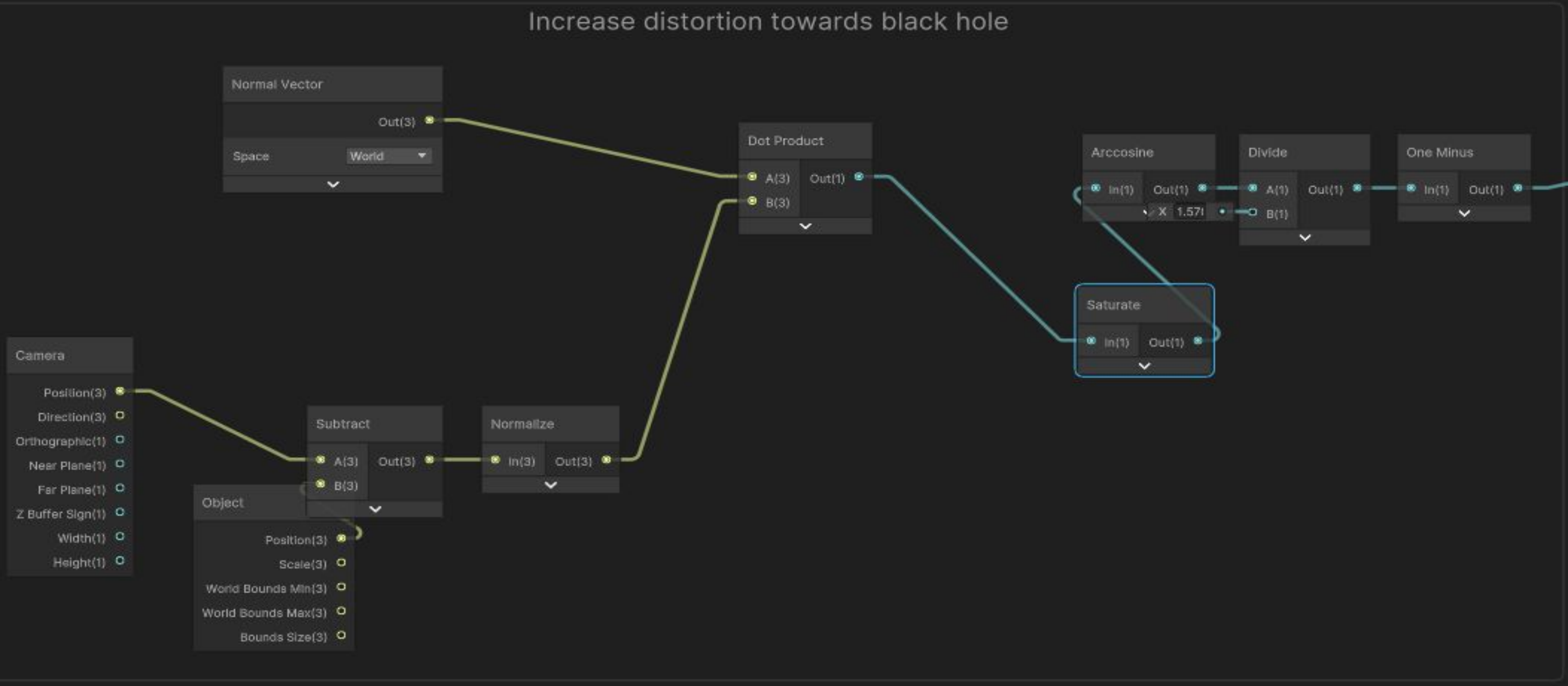
Reference background



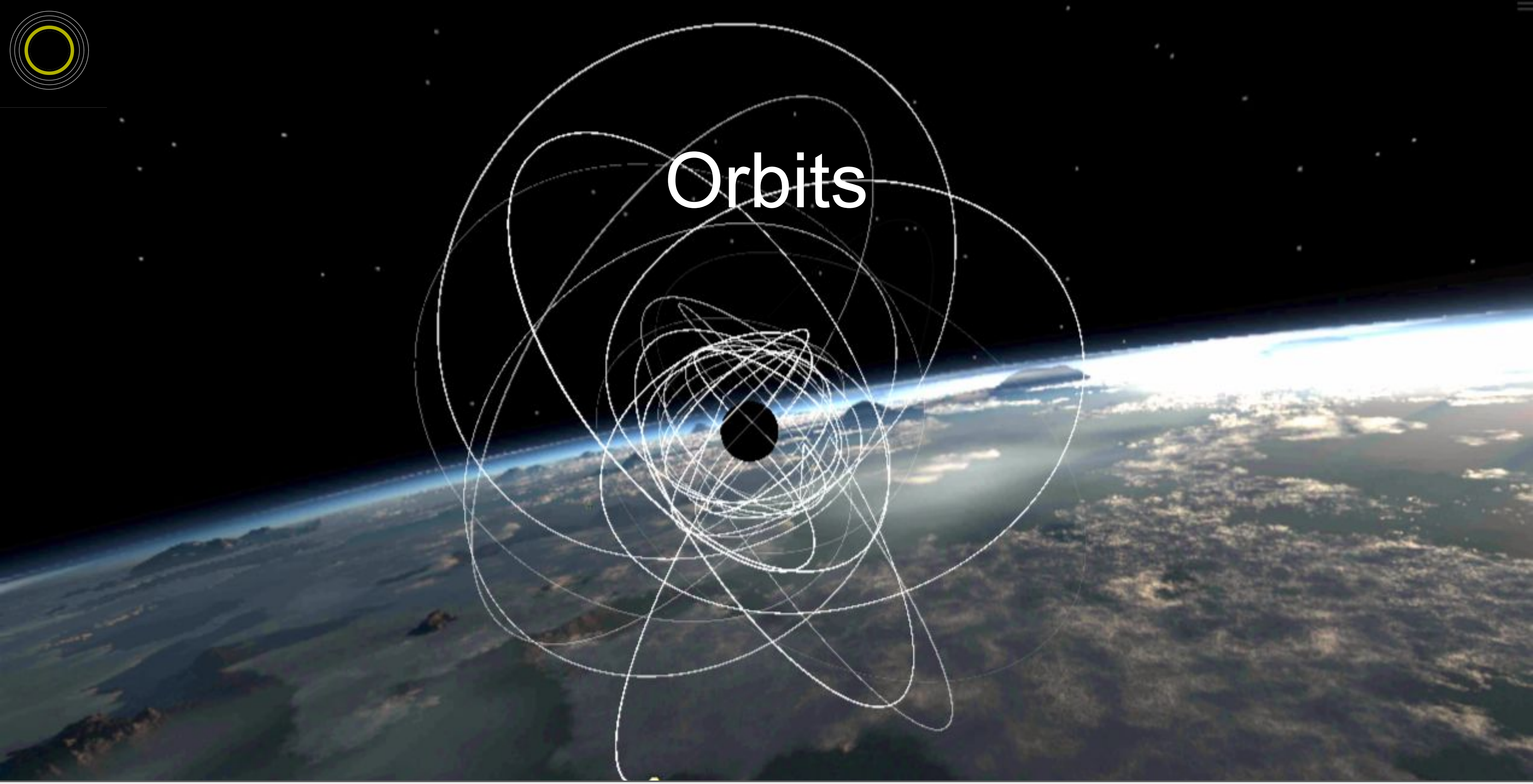
- Firstly Blackhole is a **Screen Space**
  Next the ... image is ... an intensity map


Increase distortion towards black hole
Minimize distortion at grazing angles
Screen Space vector towards center
Blackhole Mask
Black Hole color
Fresnel glow on edges on black hole

Distortion Code



Orbits



Orbits

- **Time-like** geodesic around a **Kerr** Black Hole.
- Equations of motion found by considering Lagrangian and Euler-Lagrange equations

$$\left(\frac{dr}{d\lambda}\right)^2 = [E(r^2+a^2) - aL]^2 - \Delta[r^2 + (L-aE)^2 + Q],$$

$$\left(\frac{d\theta}{d\lambda}\right)^2 = Q - cot^2\theta L^2 - a^2 cos^2\theta(1-E^2),$$

$$\left(\frac{d\phi}{d\lambda}\right)^2 = csc^2\theta L + aE\left(\frac{r^2+a^2}{\Delta} - 1\right) - \frac{a^2 L}{\Delta},$$

$$\left(\frac{dt}{d\lambda}\right)^2 = E\left[\frac{(r^2+a^2)^2}{\Delta} - a^2 sin^2\theta\right] + aL\left(1 - \frac{r^2+a^2}{\Delta}\right),$$

$$\dot{x}^\mu \dot{x}_\mu = \begin{cases} -1 & \text{for timelike geodesics} \\ 0 & \text{for null geodesics} \\ 1 & \text{for spacelike geodesics} \end{cases},$$

- The constants of motion **E** (specific energy), **L** (angular momentum - z component) and **Q** (Carter constant)
- Equations of motion reparameterized in Mino time $\lambda = \int d\tau / (r^2 + a^2\cos\theta)$

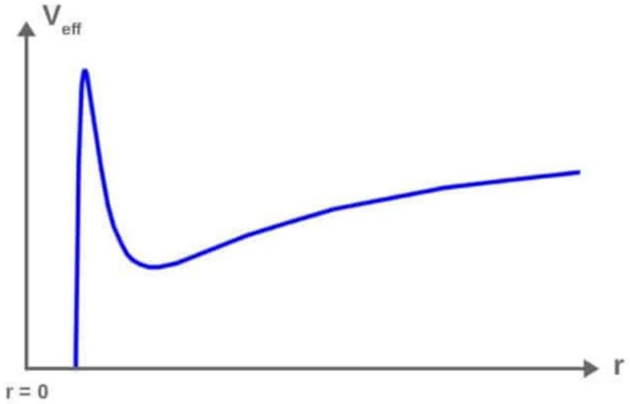
Bound and Unbound Orbits

- **p** - (semi-latus rectum), **e** - (eccentricity), **θ□¡□** - (minimum polar angle)

- Newtonian → Conic cross sections
- GR → Precession ellipses, Zoom-Whirl,…

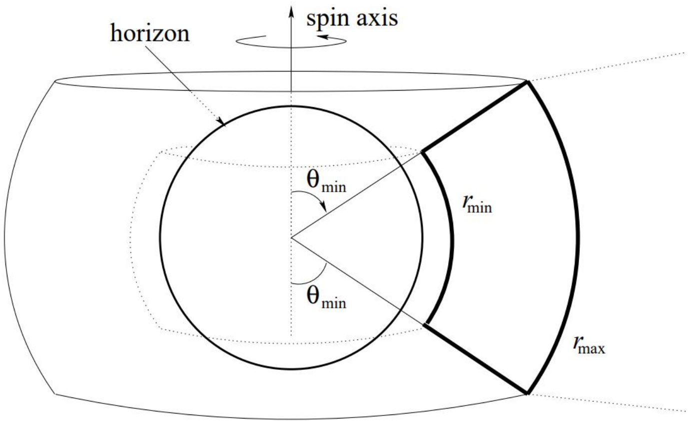
- **Effective Potential** $V_{eff}\left(r\right) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{mc^2r^3}$

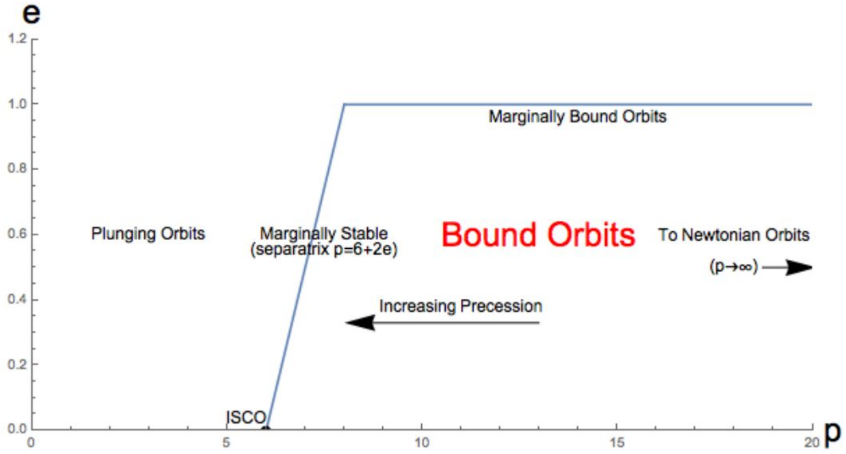
- Innermost Stable Circular Orbit (ISCO)
- Innermost Bound Circular Orbit (IBSO)



Zoom Whirl Effect

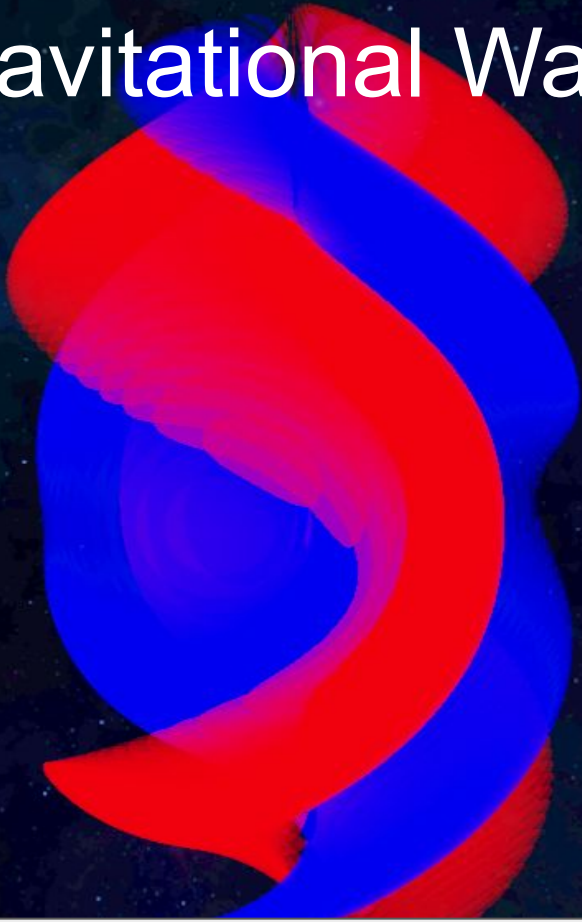
- Particle undergoes a **Zoom Whirl** effect caused by a high rate of precession.





- Parameters *p, e* and *a* are near the **separatrix** - line in parameter space that separates plunging and stable orbits.

Gravitational Waves



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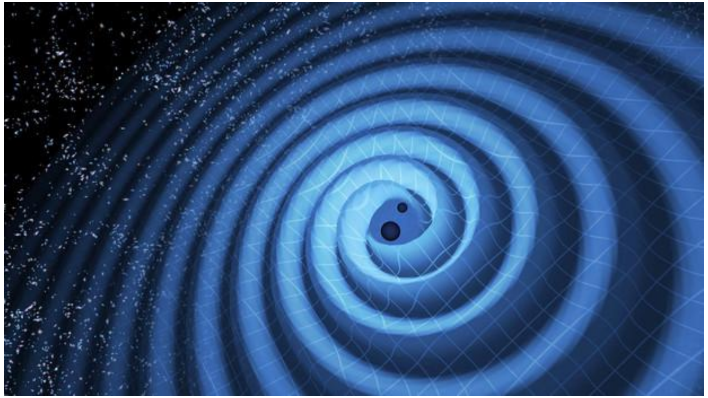
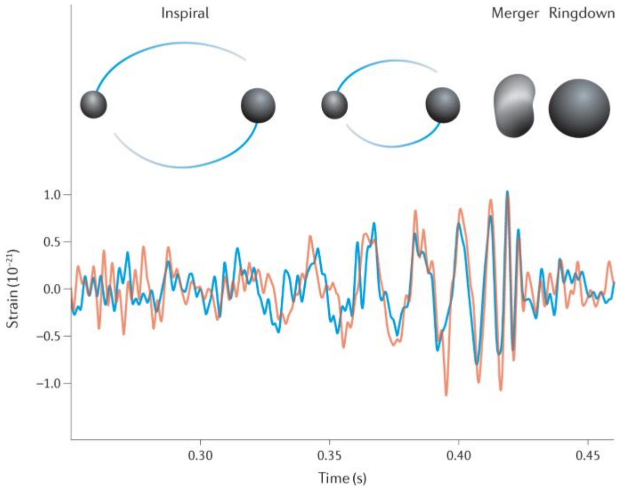
What are Gravitational Waves?



- **Ripples** in spacetime caused by the acceleration of **massive bodies**
- Ripples → **Waves** that propagate outwards at speed of light



- Waves distort spacetime and distances between objects oscillates
- Binary Black holes and Neutron stars are good sources

How do we describe Gravitational Waves?

- Detect Gravitational Waves in **Nearly Flat** Spacetime

$$g_{ab} = \eta_{ab} + h_{ab}, \qquad ||h_{ab}|| \ll 1 .$$

- **Weak Field** Einstein equations (linearised gravity)

$$\Box \bar{h}_{ab} = -16\pi T_{ab}$$

- GR Waves → **Homogeneous** solution: T = 0

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) \bar{h}^{\alpha\beta} = 0.$$

$$\bar{h}^{\alpha\beta} = A^{\alpha\beta} \exp\left(ik_\alpha x^\alpha\right),$$

- **Wave equation** travelling at speed of **light**

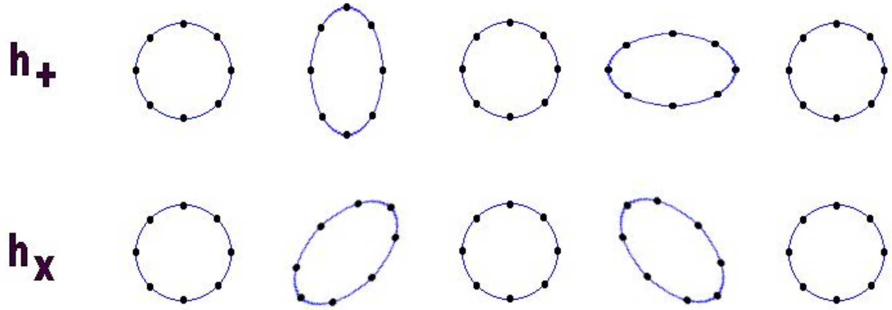
Polarization

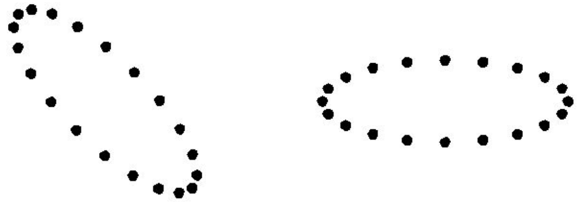
- By imposing gauge freedoms we can transform $A_{\alpha\beta}$ to the **Transverse-Traceless** gauge

$$
(A^{\mathrm{TT}}_{\alpha\beta}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

- Polarised have **two** different **polarization components**

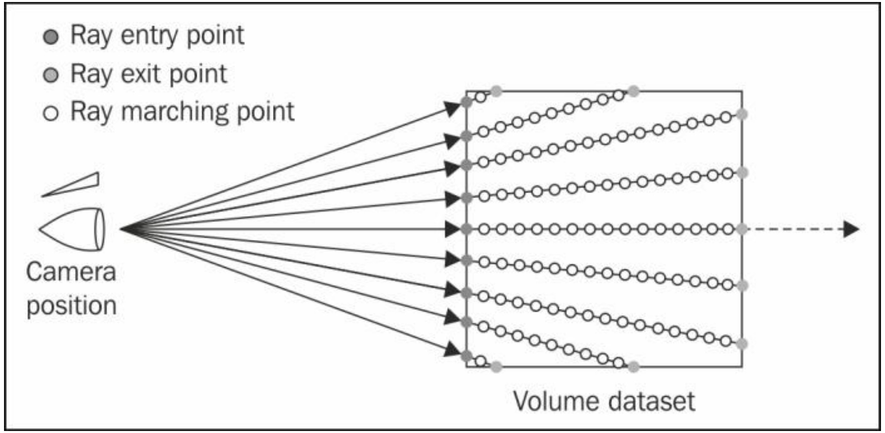
$$
h^{\mathrm{TT}}_{xx} = -h^{\mathrm{TT}}_{yy} \equiv h_+(t-z)\,; \\
h^{\mathrm{TT}}_{xy} = h^{\mathrm{TT}}_{yx} \equiv h_\times(t-z)\,.
$$





Volumetric Render

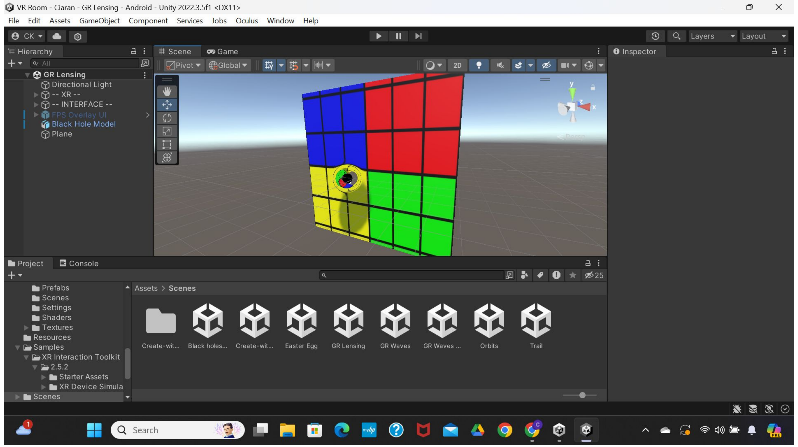
- Uses data from **SXS** (simulating eXtreme Spacetime) catalogue
- Import **mod(h)** and **Arg(h)** and interpolate between them



- **Ray-Marching technique** used
- At each step value of **h** calculated in game object
- Mod(h) determines contribution to **opacity**, Arg(h) determining **red/blue color**

Special Thanks

- VR project was done through Unity - game development software
- Many previous simulations developed by past UCD undergraduate students,
- Thanks to my supervisor Dr. Christiana Pantelidou



Dr. Phillip Lynch | Dr. Josh Mathews | Kevin Cunningham | Dr. Christiana Pantelidou

Thank you for your attention!